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THE RELATION BETWEEN LIGHTNING DISCHARGES AND ATMOSPHERICS IN RADIO RECEIVING.

BY

HARALD NORINDER, Ph.D.,

Institute of High Tension Research, University of Uppsala, Sweden.

In some of the Author's earlier papers [1, 2] the relations between lightning discharges and atmospherics in radio receiving were briefly considered. In those papers it was not possible to treat the problem in detail because of insufficient data on atmospherics. During the past two years this situation has quite altered and a very extensive number of atmospherics has been recorded at the Institute of High Tension Research of the University of Uppsala. Consequently it is now possible to discuss more in detail the relations between atmospherics and lightning discharges.

At an earlier stage of the work it was not possible to take exact records of the atmospherics. The reason for this was the same as was valid with regard to an analysis of the character of lightning discharges. For such researches we had no suitable measuring methods and instruments at our disposal. The author has already, in publications cited below,

[1] "Some Electrophysical Conditions Determining Lightning Surges." JOURNAL OF THE FRANKLIN INSTITUTE, 205, No. 6, June 1928.

[2] "Die Blitzentladungen als Ursache atmosphärischer Rundfunkstörungen." *Elektrische-Nachrichten-Technik*, 9, No. 6, Berlin, 1932.

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explained how this limitation of methods was avoided by the application of cathode ray oscillographic recording methods on the lightning discharge problem. The same must obviously be the case as regards a research of atmospherics.

A first glance at the problem of atmospherics clearly shows that they are caused by electric discharge procedures within the free atmosphere. From a physical point of view we must thus suppose the atmospherics to contain discharge procedures, which at times are quite similar to lightning discharges. From this aspect of the problem it seemed quite reasonable to try and analyze the relations between lightning discharges and atmospherics.

1. THE METHODS OF RECORDING ATMOSPHERICS AND LIGHTNING DISCHARGES.

A. The Basis of Observation.

The atmospherics have been studied with a method similar to the one used in connection with the Author's investigation of lightning discharges as described earlier.[3] The atmospherics were recorded by using horizontal aerials in connection with resistances and cathode ray oscillographs of the Author's [4] construction. The measuring methods consisted in an oscillographic recording of the voltage variations produced by the atmospherics above a resistance in the horizontal aerial. The variations of the electric field, as produced by the atmospherics, were calculated either by an integration procedure or in a simplified manner. With the field force E expressed in volt/meter, the antenna height h in meter, the measuring and damping resistance R in ohms, the capacity of the system C in Farad, the oscillographic voltage V in volts we have the relation (1):

$$\frac{dE}{dt} = \frac{1}{h}\left\{\frac{dV}{dt} + \frac{1}{Rc}V\right\}. \qquad (1)$$

From relation (1) follows the relations (2) and (3):

$$\frac{dE}{dt} = \frac{1}{hRc}V; \qquad E = \frac{1}{hRc}\int V dt, \qquad (2)$$

[3] "The Nature of Lightning Discharges." JOURNAL OF THE FRANKLIN INSTITUTE, 218, No. 6, December 1934.

[4] "Ein besonderer Typus des Kathodenoscillographen." *Zeitschrift für Physik*, 63, 9–10, Berlin, 1930.

$$\frac{dE}{dt} = \frac{1}{h}\frac{dV}{dt}; \qquad E = \frac{1}{h}V. \tag{3}$$

A small value of R is necessary for the approximation as in relation (2) and the method is called the dE/dt method. A high value of R is necessary for a corresponding approximation as in (3) and this method is called the E-method. The use of the dE/dt method allowed, in general, a recording of atmospherics of short duration with values from 100 to 600μ sec. These types of atmospherics were much more usual than atmospherics of long duration with times between 2000 to 6000μ sec. These longer atmospherics were observed by using the E-method.

The difference between the measurements of the variations of the field force, as produced by lightning discharges and compared with the field force variations of atmospherics consisted, mainly, in a difference of the incoming voltage amplitudes above the resistance R from the aerials in the two cases. At lightning discharges the highest voltage values above R were observed to be of the order of 1000 volts, at atmospherics of usual amplitudes the corresponding values above R were of the order of one volt or smaller values. In the latter case it was necessary to apply an amplifying set between the aerial system and the cathode ray oscillograph. The amplifying set had obviously to be arranged so that it excluded a distortion of the impulses coming from the aerial. This feature of the arrangements was controlled by tests whereby we used impulses of a variation form similar to those occurring in atmospherics.

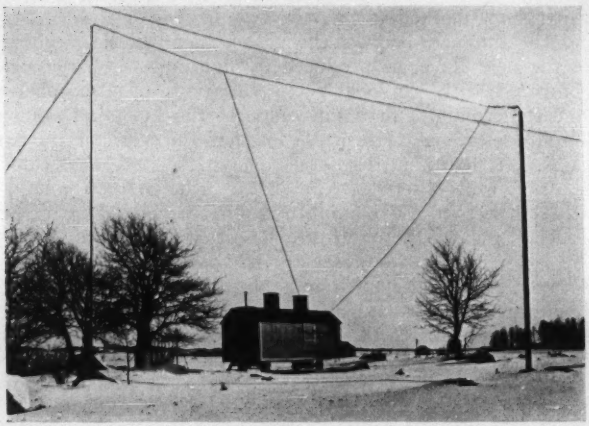
B. External Arrangements.

The atmospherics were observed in field stations of special design constructed at the Institute. The stations were easily moved from one observation place to another. This was quite necessary as regards our preliminary observations. Here it was necessary to observe to what extent a vicinity between the observation station and high tension transmission lines caused a disturbation of the values. The movable field stations were also necessary and useful in the case of simultaneous observations between two stations.

A picture of such a station is reproduced in Fig. 1. Of this type of station we used two, both equipped with cathode ray oscillographs, amplifying sets and necessary auxiliary apparatus. An interior of such a station is reproduced in Fig. 2.

The necessary amplification was sometimes as high as 800 times with regard to the incoming values, and this necessitated the location of the field station at distances as far as possible from ordinary transmission power lines. It was impossible to use advantageously the electric power of the station supplied

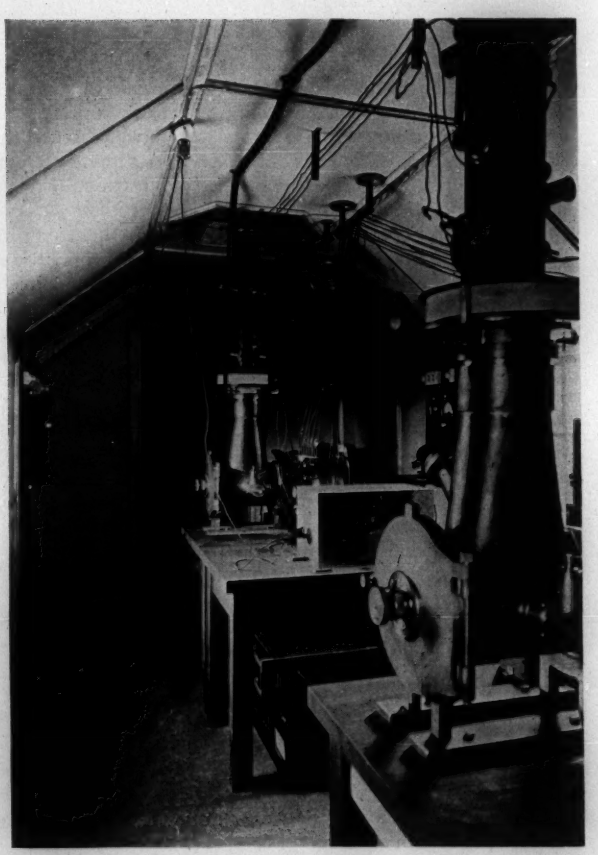
FIG. 1.



Movable field station for recording atmospherics.

either by earth cables or aerial lines because such systems very often provoked disturbances. This difficulty was avoided by using a transportable generator driven by a petrol motor. This arrangement had many advantages and allowed the station to be put into operation within a few seconds. Another advantage of these movable generators was their safe operation during heavy thunderstorms. This was of especial importance because a connection of the field station to an ordinary electric power line very often gave rise to interrup-

FIG. 2.



Installation of cathode ray oscillographs in a field station.

tions in connection with thunderstorms, and especially when our observations were of greatest importance.

C. The Observations.

In order to examine the relations between lightning discharges and atmospherics we used different methods. In such situations when the lightning discharges were developed within a few kilometers from the observation station we recorded the field variations direct from the antenna without any amplifier, a procedure already described in another publication.[5] When such a thunderstorm passed over to greater distances we continued to observe the field variations of the lightning by the inter-connection of an amplifier. At times operations were also carried out by using telephonic messages in connection with the existence of a thunderstorm at a place sufficiently distant, and thus recorded the occurring atmospherics of which a predominant part were of a very characteristic type—the lightning type.

Another very useful method was the following. We connected an ordinary broadcasting receiving apparatus to an aerial when characteristic loud disturbances as caused by atmospherics in the receivers were expected to be audible. Together with this arrangement cathode ray oscillographs were in operation and connected above amplifiers to an aerial. The method yielded a simultaneous record of such atmospherics which were observed to provoke loud disturbances in the radio receiver.

Apart from these methods we have carried out a more general investigation into atmospherics. These observations covered a period of two years. Usually, oscillograms were taken twice a day—during the day time and during the night. The total number of oscillograms of atmospherics was about 22,000 during an observation period ending in December 1935. The variation forms observed when making these extensive records were very often so similar that we found it sufficient to limit the calculation by an integration procedure to only 10 per cent of oscillographic records taken. The result of this treatment was found to be quite sufficient as regards the obtaining of the characteristic properties of the atmos-

[5] Loc. cit.

pherics. The author has published his first results with regard to the nature of atmospherics elsewhere.[6, 7, 8] Since the publication of the Paper mentioned extensive data has been collected which deals more intimately with the relation between atmospherics and lightning discharges—a problem that will be considered here.

2. THE VARIATION FORMS OF AUDIBLE ATMOSPHERICS.

When we were listening into an ordinary radio receiving set we easily distinguished two characteristic loud-forms of perturbations as caused by atmospherics. The one was very short and distinct and classified as the loud-type "clicks." The other type, which was of long duration and produced a whistling or loud hissing, was characterised as "grinders." This classification of the loud type is in accordance with what has been observed earlier and pointed out in other publications which deal with the problem of atmospherics. Both types have been observed to exist in connection with thunderstorms. Our methods have allowed us, by using the cathode ray oscillographs, to analyze in a most exact manner the variation forms of these two characteristic types of atmospherics.

A. The "Clicks" Atmospherics of Short Duration—Their Origin.

Some original oscillograms illustrating the atmospherics of the "clicks" type are reproduced in Fig. 3. Perturbations of this type are characterised by non-complicated variation forms. Furthermore, it is easy to reproduce this perturbation form in an artificial manner by using an impulse generator for high tension. In such a case the generator can be regulated to produce impulses of a unipolar or a bipolar type with a total duration of the impulse of e.g. 50 to 200μ sec. The impulses are impressed upon a sending antenna circuit, and within a certain region it is possible on receiving antennas connected to

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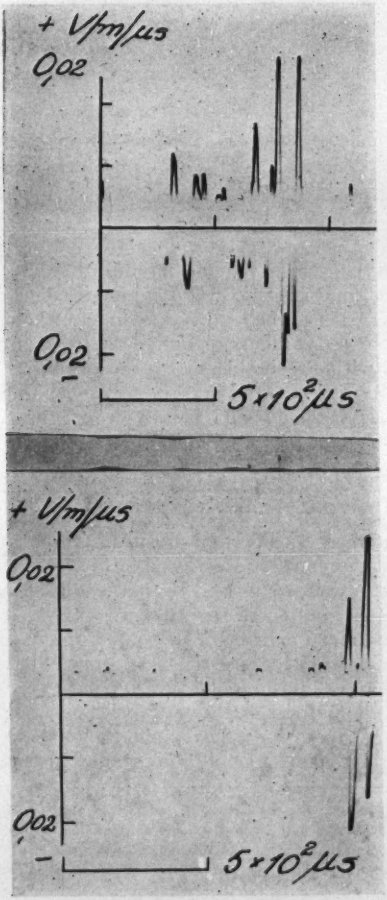
[6] "Untersuchungen von Blitzentladungen und atmosphärischen Rundfunkstörungen in Schweden mit dem Kathodenstrahloszillographen." *Elektrotechnische Zeitschrift*, **56**, No. 14, Berlin, 1935.

[7] "Undersökningar över de atmosfäriska rundradiostörningarnes natur." *Tekniska Meddelanden från Kungl. Telegrafstyrelsen*, No. 7–8, Stockholm, 1935.

[8] "Cathode Ray Oscillographic Investigations on Atmospherics." *Proceedings of the Institute of Radio Engineers*, **24**, No. 2, New York, 1936.

FIG. 3.



Oscillograms of atmospherics of the "clicks" type.

ordinary receivers to observe disturbances of the above mentioned characteristic "clicks" type.

It was of special interest to search for the proper origin of these types of atmospherics. In some cases we have, by direct observations in thunderstorms, found that this type of atmospherics was caused apparently by very short discharges within adjacent parts of the thunderstorm clouds. We do not consider such short discharge types within the thunderstorms as lightnings in the proper sense of the word but more as local sparks. We have sometimes observed these local sparks to be a secondary discharge phenomena caused by a heavier lightning discharge. They are, in such cases, observed some moments after the disappearance of the ordinary lightning. We presume that they are caused by limited secondary displacements of the charges from an ordinary lightning.

It is of interest, in this connection, to discuss some general questions with regard to lightning discharges. A lightning is accepted as a spark phenomena of very extensive proportions. This interpretation of lightning is a directive and is of extreme importance with regard to the electrophysical conditions of lightning.

A thunderstorm, from the standpoint of a meteorologist, is defined as a procedure in the atmosphere which is connected with disruptive discharges that ought to be visible or audible or both. Such discharges—the ordinary lightning—afford, for their development, charge accumulations and field forces of dimensions along the discharge region that we are not yet able to reproduce and operate in our laboratories.

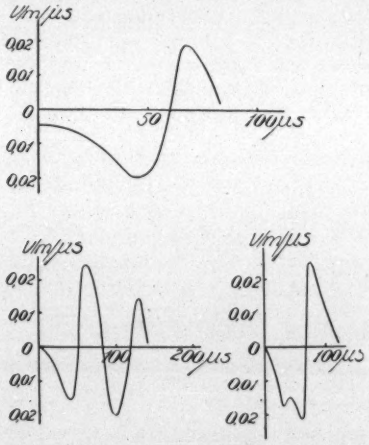
The accumulation of charges and the strong fields are thus defined as a thunderstorm when lightning has occurred within the active regions of the thunderstorm atmosphere. With regard to the electric stress within the thunderstorm cloud there must on the other hand at an earlier stage of the discharge procedure exist fields of an intensity high enough to produce local shorter sparks and brush discharges before the proper discharge limit of lightning is reached. These discharges caused by local and shorter sparks are not easy to detect by visual observations from the earth because their intensity is a low one. They are visible if we are able to observe them closely, as for instance, from an airplane travers-

ing local showers of rain, snow or hail. My opinion is that, some of the atmospherics have their origin from field variations caused by such local discharges; and this is especially the case with certain of the type classified as "clicks."

There is a distinct difference between the variation character of the "clicks" in comparison with ordinary lightnings. The latter consist of repeated groups of discharges covering relatively long time intervals. The clicks on the other hand do not consist of such repeated discharges. This singular character of the "clicks" is very marked and is an indication of the special character of these discharge forms.

Some typical oscillograms of "clicks" are reproduced in a linear time scale in Fig. 4. Integrated curves are reproduced

FIG. 4.



Oscillograms of atmospherics of the "clicks" type in linear time scale.

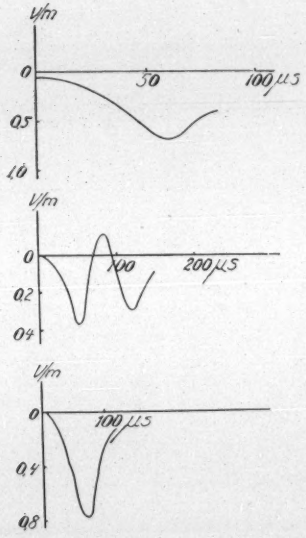
in Fig. 5. The curves of Fig. 5 have a relatively high amplitude in volt/meter as compared with ordinary atmospherics. This indicates that they have not passed any long distance from their source of origin prior to being taken up by the antenna system. Compared with ordinary lightnings their

initial variation amplitude is on the other hand relatively small.

B. The "Grinders" Atmospherics of Long Duration—Their Origin.

The "grinders" are quite a contrast to the "clicks" with regard to their general aspect and variation forms. The "grinders" are developed over time intervals of the same order

FIG. 5.

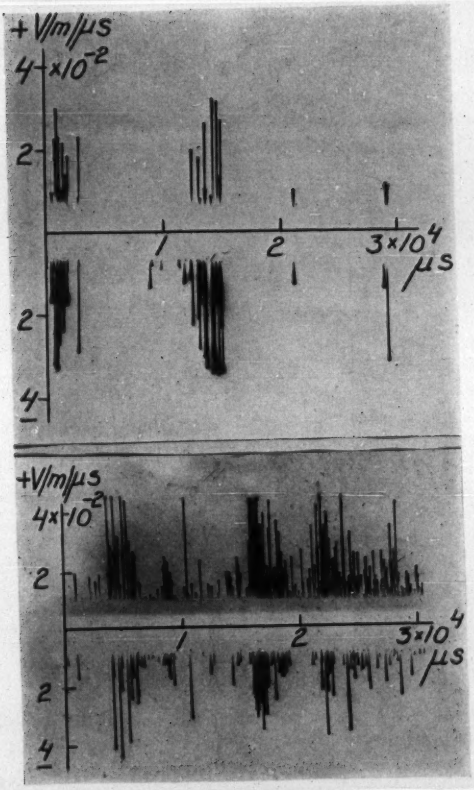


Calculated field variations of atmospherics of the clicks type.

of duration as we have observed to be valid for ordinary repeated lightnings. We have in Fig. 6 reproduced some typical original oscillograms of "grinders" and from which it is evident that they consist of repeated swarms of field variations extended over long time intervals.

We have every reason to believe that "grinders" are caused by distant lightning discharges. From this point of view it is of special interest to compare them with the general variation

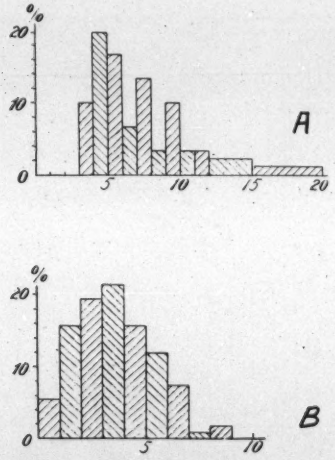
FIG. 6.



Oscillograms of atmospherics of the "grinders" type.

structure of lightning. We have, in this respect, in Fig. 7, compared the percentage number of partial groups of discharges in "grinders" A with corresponding groups of partial discharges in lightning B as have been observed earlier. The relative number of partial discharges is slightly higher in the "grinders" when compared with the values of lightning. This difference is caused by the impossibility, when recording "grinders," to avoid stray atmospherics of the short variation

FIG. 7.



Percentage number of partial discharges in "grinders" (A) compared with number of partial discharges in lightning (B).

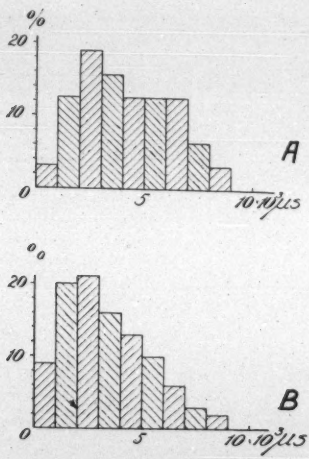
type interfering with the records when recording "grinders." With regard to this interference we must regard the congruity between the two groups as being satisfactory.

The total duration of grinders A compared with the corresponding duration of lightning discharges B is shown in Fig. 8, from which a good congruity between the groups will be seen.

Another method for comparison has also been used. If a thunderstorm has passed far away from the observation

locality it has sometimes, under favourable conditions, as already mentioned, been possible to indicate the lightning discharges by a telephonic message, or by other methods. Corresponding oscillographic records of atmospherics have resulted in a special type which has been classified as the lightning type of atmospherics. We have in Figs. 9–10 exemplified some oscillograms of such types. The congruity with the "grinders" is evident.

FIG. 8.



The total duration of atmospherics of the "grinders" type (*A*) compared with the corresponding total duration of lightning discharges (*B*).

3. ATMOSPHERICS OF ORDINARY TYPE AND THEIR RELATION TO LIGHTNING DISCHARGES.

A. The Usual Type of Atmospherics.

As already mentioned we observed atmospherics under such undisturbed conditions when no local thunderstorms or similar disturbances influenced the results. Two main types of atmospherics were obtained during this investigation, (*a*) atmospherics of short duration with a most frequent occurrence of a duration time of 200–300 μ sec. or 300–400 μ

FIG. 9.

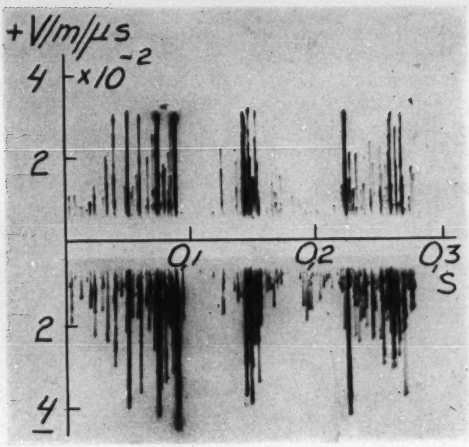
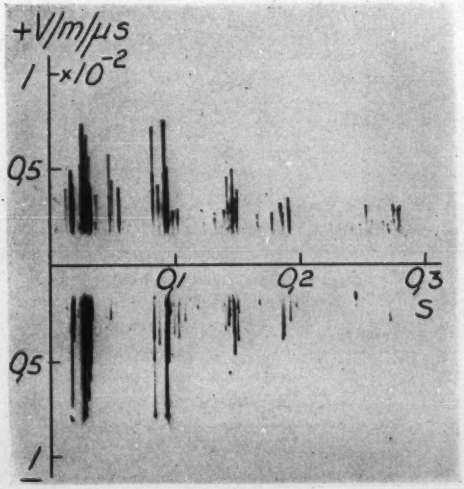
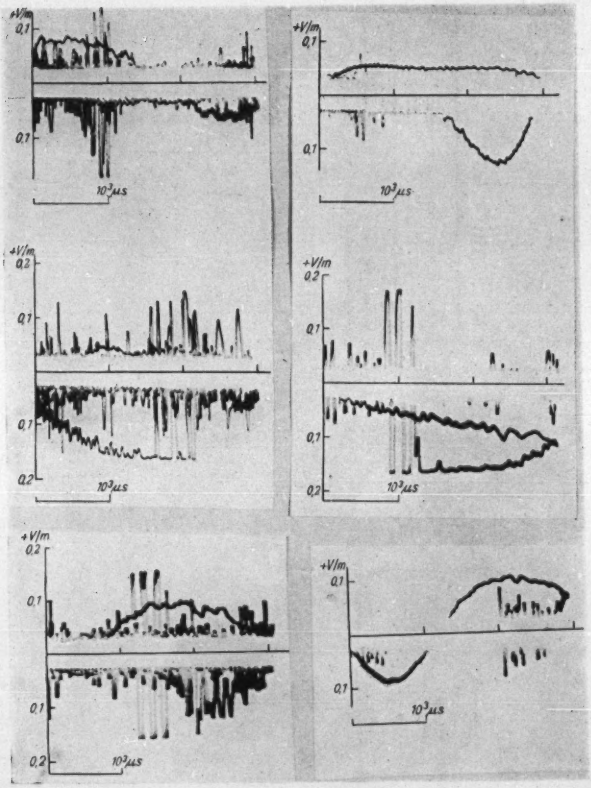


FIG. 10.



Atmospherics of the lightning type.

FIG. 11.

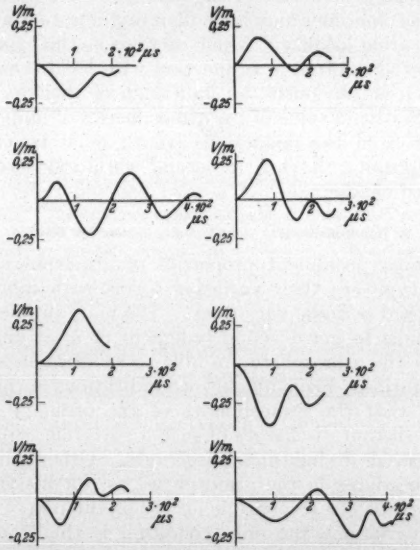


Atmospherics of the long duration type.

sec., (*b*) atmospherics of long duration with a time of 3000–4000 μ sec. or 4000–5000 μ sec. as the most occurrent values. In comparison with the corresponding short duration types the atmospherics of long duration occurred in only a few per cent. of cases during simultaneous observation periods.

Original oscillograms of atmospherics of the long duration type are exemplified in Fig. 11. The atmospherics have been recorded with the *E*-method which allows a direct calculation of the values of the field force.[9] The types are characterized at times by a very marked superposition of variation forms of short duration. Some of the atmospherics of the long duration type are, without doubt, caused by lightning discharges.

FIG. 12.



Variation forms of atmospherics of the short duration type.

[9] H. Norinder und R. Nordell: "Vergleichende Untersuchungen von Rundfunkstörungen von längerer und kurzerer Dauer." *Elektrische Nachrichten-Technik*, 12, No. 10, Berlin, 1935.

The atmospherics of the short duration type have been observed to a very great extent. The general congruity between individual oscillograms enabled us, as has already been mentioned, to limit the proper calculation by an integration procedure to comprise only 10 per cent. of the oscillographed curves, which in this case consisted of dE/dt curves.

From the whole sample of calculated atmospherics of the short variation type we have selected some typical variation forms which have been reproduced in Fig. 12.

An examination of Fig. 12 shows that one type, the unipolar one, is different from the others which are bipolar and have more or less typical superimposed variations of a periodic character. The unipolar types are in some cases very similar to the variation structure of the "clicks." We presume that such types sometimes may have their origin not far away from the observation locality. Much more so as the type in question occurs very often in connection with local shower situations which is, obviously, an indication of their local origin. On the other hand some of the atmospherics of unipolar type, as follows from the deductions below, must be caused by distant lightning having a special variation form of the transported charge.

B. Transformation of an Initial Lightning Impulse with Distance.

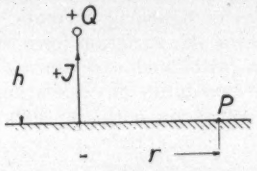
The most prominent properties of atmospherics of the ordinary type are their variation forms with more or less superimposed periodic variations. The main problem in this connection is to get a sound judgment of what kind of discharges in the atmosphere are able to produce these typical variation forms. From all our own indications in this respect it follows that the atmospherics of the ordinary type are caused by distant discharges of such a variation type as have been observed in lightning discharges. Other similar discharge procedures in the atmosphere, as, for instance, local sparks, may also be responsible for the production of the same types of variation—the only condition is that their initial intensity is high enough.

An impulse emitted from a lightning path is mainly governed by the same laws as are valid for the transmission of radio waves. The consequence of this must be obvious and

the distance between the centre of emission and the receiving station will, in this respect, be a selective factor of importance. Only a few of all the impulses emitted from a lightning path will, at a greater distance, reach a receiving station with amplitudes sufficiently high enough to be recorded with ordinary amplifying arrangements. This presumes a pronounced amplitude variation in partial lightning discharges. Such a graduated variation has often been observed to occur in lightning discharges, and just in this connection, the measurements of the variations caused by lightning discharges in the magnetic field are very indicative.

The main problem to be solved in this connection is the transformation of a given variation form of an impulse with

FIG. 13.



Location of a lightning charge for calculation.

the distance from the centre of emission. We have in this respect to calculate with the fundamental equations of Maxwell for transmission of electric impulses, presuming, of course, that we can in this first approximation neglect the influence of the damping, when the waves pass through ionized layers. General deductions of the equations of the transmission of an impulse are given by P. Lejay.[10] This author lacked observational data for direct application to a calculation of the transformations when impulses passed to distances from the source of emission.

The calculation of the transformation of an impulse with the distance from the discharge centre—e.g. the lightning path, has in our case been considered to be limited to a vertical [11] partial lightning transporting a charge Q (see Fig. 13) from the

[10] P. Lejay: Les perturbations orageuses du champ électrique et leur propagation à grande distance. Inagural diss., Paris, 1926.

[11] It is also possible to apply the calculation to discharges between two points on the free atmosphere but we have not considered such a calculation necessary in this connection.

height h in the atmosphere to earth. The current value is supposed to be I, the horizontal distance r where $r \gg h$. Under such presumptions it is possible to deduce the relation (4) with regard to the electric field force E_r and its dependance of the distance:

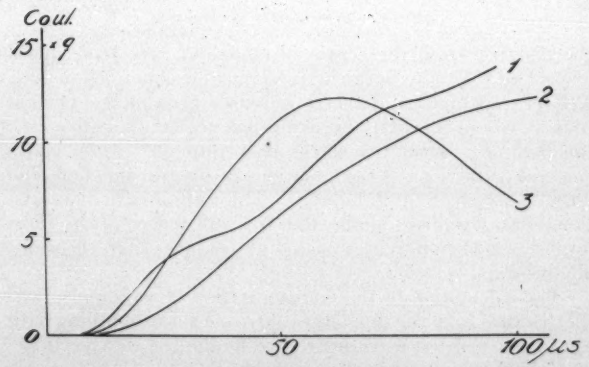
$$E_r = \frac{2h}{r}\left\{\frac{c^2}{r^2}Q + \frac{c}{r}I + \frac{dI}{dt}\right\}. \qquad (4)$$

All units are in e.m.u. and with consideration to $I = dQ/dt$ we receive the relation (5):

$$E_r = \frac{2h}{r} \cdot 10^{-7} \cdot \left\{\frac{9.10^{10}}{r^2}Q + \frac{3.10^5}{r}\frac{dQ}{dt} + \frac{d^2Q}{dt^2}\right\}. \qquad (5)$$

From the Author's cathode-ray oscillographic records of the variation forms of lightning currents [12] there are possibilities of determining the variation form of Q. An application of the relation (5) to such experimental variation curves thus results in the possibility of calculating the variation of the field force in volt/meter with the distance from the discharge centre.

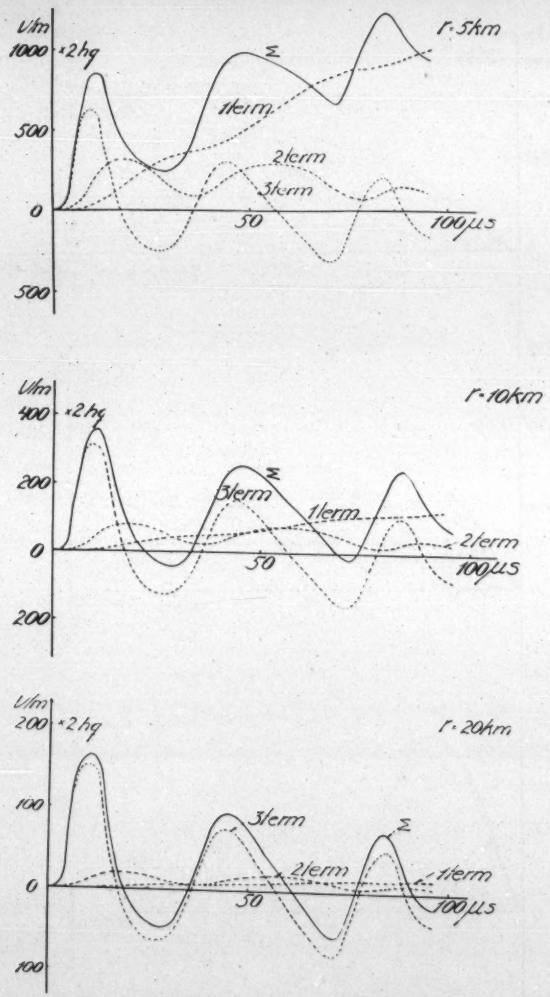
FIG. 14.



Charge variation curves in lightning.

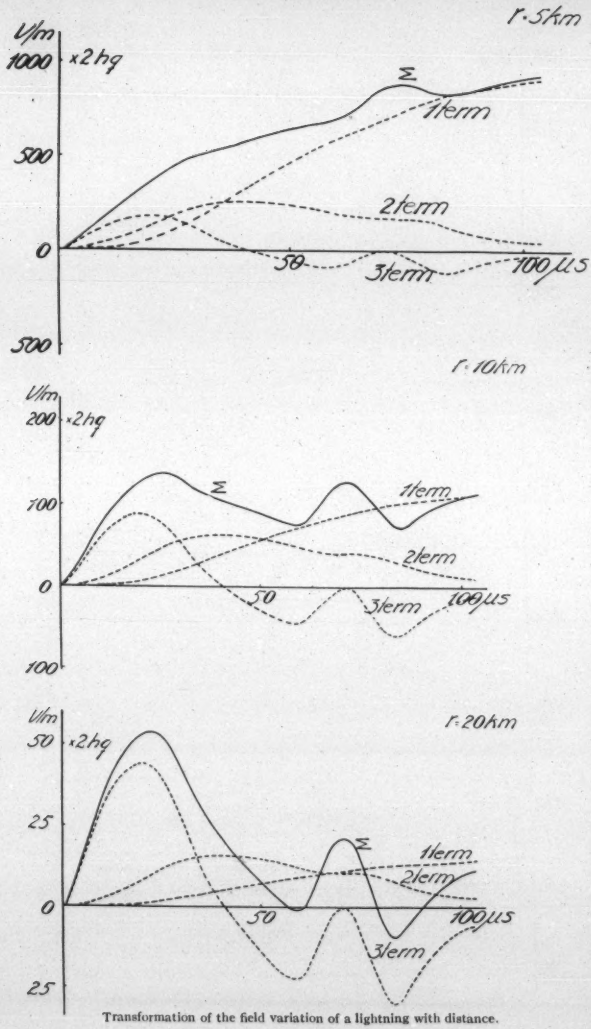
[12] "Lightning Currents and Their Variations." JOURNAL OF THE FRANKLIN INSTITUTE, 220, No. 1, June 1935.

FIG. 15



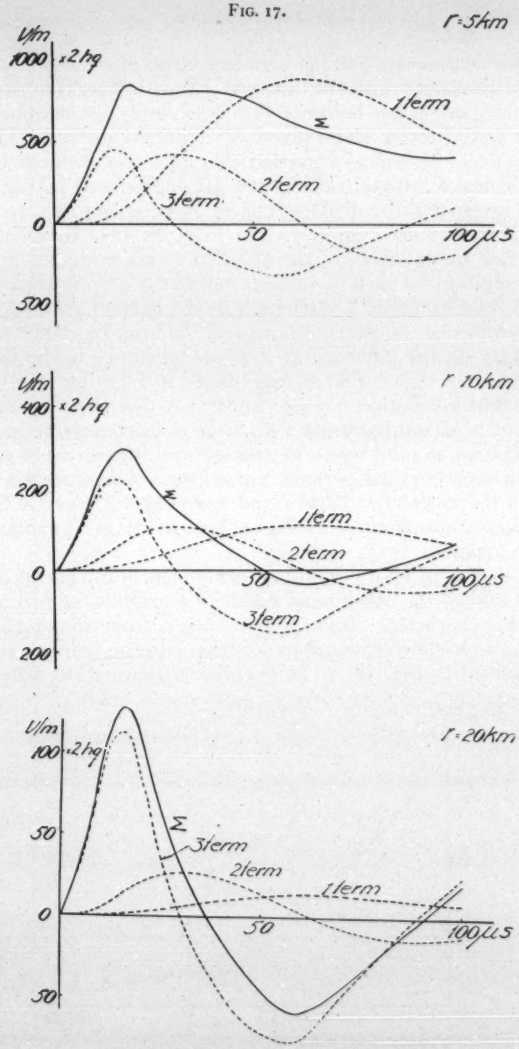
Transformation of the field variation of a lightning with distance.

FIG. 16.



Transformation of the field variation of a lightning with distance.

FIG. 17.



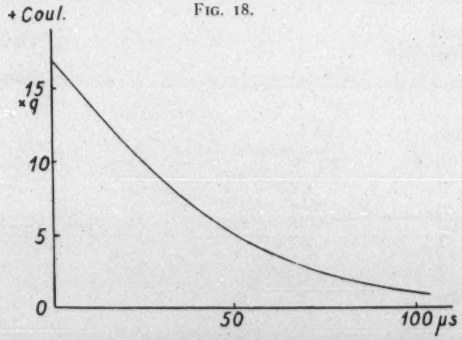
Transformation of the field variation of a lightning with distance.

Our consideration of the variation forms of atmospherics as a transformation with the distance of the discharges in partial lightning can easily be controlled if we apply the relation (4) upon our directly determined Q variation curves. Three such curves of typical variation character and deduced from our typical lightning oscillograms are reproduced in Fig. 14. The corresponding calculated curves at a distance of 5, 10 and 20 kilometers are reproduced in Figs. 15–17. In order to illustrate the influence of the different terms we have, in this case, reproduced each of these terms in the graphical curves. From relation (4) it is obvious how the current value I and dI/dt influence the transformation of the impulse. The term of preponderant influence at a greater distance is the third.

From the calculation as exemplified in Figs. 15–17 a very important conclusion can be drawn. A discharge variation form of a partial lightning will, after a short passage in the atmosphere, in most cases, be transformed into forms of more or less superimposed periodic variations. A comparison between the calculated curves and the observed curves from ordinary atmospherics (see Fig. 12) results in good congruity of the types.

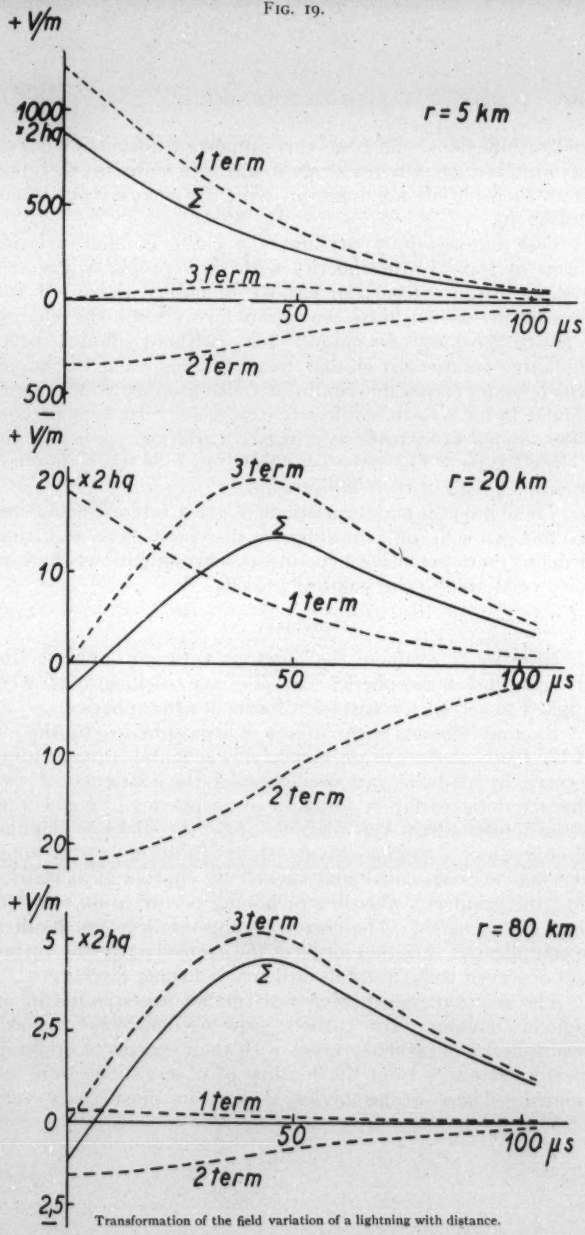
A special and not very usual form of the initial Q variation curve can on the other hand result in transmission curves of unipolar character. Such a curve taken from the Author's oscillographic determinations of the current variations is exemplified in Fig. 18. The Q curve is characterized by its

FIG. 18.



Charge variation curve of aperiodic type of lightning.

FIG. 19.



Transformation of the field variation of a lightning with distance.

lack of inflexions, and from this property transmission curves of unipolar type will obviously follow at a sufficient distance. How such curves are built up with distance is exemplified in Fig. 19.

Our analysis fully explains the characteristic variation forms of typical atmospherics with their proper origin at a greater distance from the locality of observations. It was possible to deduce these variation forms from the characteristic lightning discharges, but, without doubt, other discharge forms exist similar to partial lightning discharges which, under favourable conditions, also produce atmospherics similar in form to those already explained. To such we consider under favourable conditions variation forms of the "click" type. The only condition must be that they are intense enough in their initial state.

There are still many questions of great interest connected to the problem of atmospherics discussed here but they demand more extensive experimental investigations than we have been able to carry out at present.

SUMMARY.

In order to examine the relations between lightning discharges and atmospherics cathode ray oscillographs were applied to record the variation forms of atmospherics.

By simultaneous observations of atmospherics, by the aid of the loud speakers of radio receiving sets and corresponding records by cathode ray oscillographs the existence of two characteristic variation forms of atmospherics in connection with thunderstorms were obtained, (*a*) "the clicks"—singular discharges of a comparatively short duration (up to some hundred microseconds) and caused by shorter local sparks, (*b*) "the grinders" whistling or hissing perturbation forms in the loud speakers. The corresponding oscillograms resulted in complicated variation forms of the same type as the Author had observed to be usual in ordinary lightning discharges.

The relation to lightning was further investigated in an indirect manner. By cathode ray oscillographic records atmospherics of ordinary types with their centres of origin at greater distances from the locality of observations were examined. These atmospherics showed in most cases very

characteristic variation forms with more or less pronounced and superimposed periodic or quasiperiodic variation. The main problem was how to explain the generation of these variation forms from known discharges as they were observed at the most probable centres of emission—e.g. lightning paths. For that purpose the transmission equation for radio waves was applied on such discharge variation forms which had been observed to be usual in partial lightning discharges. From the calculation it was concluded that atmospherics with typical variations could be derived from lightning discharges with known and uncomplicated variation forms fixed by cathode ray oscillographic recording. The sometimes apparently complicated variation forms of atmospherics of the short variation type were thus in the most cases found to be a secondary effect caused by gradual deformation during transmission.

MINERAL COMMERCE AND INTERNATIONAL RELATIONS

By EDWARD SAMPSON

January, 1936, page 1

ERRATA

P. 5, last line, should read "30–40 feet of overburden. Nearest shovel is mining 3 feet of coal exposed by stripping."

P. 15, second line of title to illustration. "Ripples" should read "riffles."

P. 28, line 8 of text. "18" should read "19."
line 9 of text. "18" should read "19"; "93" should read "84."

P. 42, next to last line of text. "54" should read "55."

P. 54, second line of last paragraph. For 1930 read 1934.

A Rock Storage Oven for Electrical Heat.—H. J. DANA AND R. E. LYLE. (*Heating and Ventilating*, Vol. XXXIII, No. 2.) When the decision was made to electrify Mason City, the contractors' camp at the Grand Coulee Dam Site, opportunity was given to the State College of Washington to use it as a field laboratory and to carry on an extensive series of studies relating to the use of electricity for domestic heating. One house was selected to study the feasibility of heat storage. The basement was 20 ft. by 28 ft. and a system of ducts and registers was laid out similar to that which would be employed in a warm air furnace installation. In place of the furnace, however, an insulated brick oven was built, about the same size as a domestic furnace. The space inside the oven was filled with granite boulders ranging from 3 lb. to 60 lb. in weight. The useful heat capacity was figured to be one million B.t.u. at 600 deg. with an additional 690,000 B.t.u. available by raising the temperature to 900 deg. Provision is made for inserting electric heater elements through a duct underneath the mass of rock so that, when electricity is on, the natural air circulation will carry away the heat and communicate it to the bricks in the oven. Arrangement is made so that cold air from the rooms is mixed in the right proportion with the hot air from the oven for the house heating. Forced circulation is used, by an electric fan. To keep the oven at 555 deg. 2.65 kw. was required, which cannot all be attributed to losses, as this heat escaped into the basement. In the tests made, it was revealed that the heating elements used were too small and that greater insulation should be built into the oven. There is pointed out that this may be a method for smoothing out the 24 hour load curve of utility power systems.

R. H. O.

ON THE INSTABILITY AND RUPTURE OF DROPLETS AND BUBBLES IN STRONG ELECTRIC FIELDS.

BY

LEWI TONKS, Ph.D.,

Research Laboratory, General Electric Company, Schenectady, New York.

ABSTRACT.

An approximate quantitative theory of the equilibrium of a bubble or droplet in a uniform electric field is developed and applied to earlier experimental results. An explanation of the oscillation of bubbles in strong fields and the difference in behavior of positive and negative bubbles is explained on the basis of the discharge of electricity from points.

The relation of water drop size and electric field is of importance in connection with thunderstorms, and this has led to several investigations of the effect of strong fields on drops and bubbles. Several implications and relations of the experimental results had remained undeveloped until J. Zeleny's recent article [1] appeared. Several of the questions there discussed parallel a treatment prepared by the present author, but there are certain important differences in point of view. This analysis has, therefore, been revised so as to avoid duplication and to include a treatment of these differences.

The following phenomena are generally agreed upon by experimenters in this field:

(*A*) A hemispherical soap bubble, resting on one of two parallel condenser plates is continuously distorted by increasing field between the plates up to a certain critical field E_c at which violent oscillation of the bubble sets in, and this behavior is the same whether the bubble be positively or negatively charged.

(*B*) The positive bubble bursts at or only slightly above this critical field, whereas the negative bubble can be carried to considerably higher fields before exploding.[2]

[1] J. Zeleny, JOURN. FRANKLIN INST., 219, 659 (1935).

[2] J. J. Nolan and J. G. O'Keeffe, *Proc. Roy. Irish Acad.*, 5, 86 (1932).

(*C*) The critical field E_c varies with bubble radius r_0 according to the theoretical relation,[3]

$$E_c r_0^{1/2} = cT^{1/2}, \quad c = \text{constant}, \quad T = \text{surface tension}. \quad (1)$$

(*D*) In oscillating, at the extreme elongation, the bubble peaks up into a conical tip and ejects droplets of liquid. At the same time there is current flow.[4]

In addition:

(*E*) Nolan and O'Keeffe have observed that rapid current flow begins at lower fields from negative drops than from positive.[2]

(*F*) Macky has observed that a drop freely falling in a strong field is drawn out into a long filament at its positive end but is only slightly elongated at its negative end.[5]

(*G*) He has also shown that more liquid is ejected from a positive bubble than from a negative one at the same voltage.[4]

Eq. (1) has been derived from similarity considerations which yield no value for the constant c. By assuming that the form of the bubble in the electric field is ellipsoidal, although photographs have shown that the distorted bubbles, before instability sets in, are somewhat more pointed, an approximate theory can be developed.

Let the original radius of the bubble before distortion be r_0, let the semi-major axis of the distorted bubble be a, its semi-minor axis be b, and its eccentricity e; let the uniform impressed electric field be E_0, and let the effective surface tension be T (for a bubble film with its two surfaces $T = 2T_{\text{liquid}}$). Further, let R_{bb} and R_{ab} be the two principal radii of curvature of the ellipsoid at its diametral plane (i.e., at the plane electrode supporting the bubble) and R_a be the radius of curvature at the vertex of the bubble.

Since the volume of the bubble is constant during distortion, we have

$$ab^2 = r_0^3. \quad (2)$$

Since the pressure on the enclosed fluid is the same at

[3] C. T. R. Wilson and G. I. Taylor, *Proc. Camb. Phil. Soc.*, **22**, 728 (1925).
[4] W. A. Macky, *Proc. Camb. Phil. Soc.*, **26**, 421 (1930).
[5] W. A. Macky, *Proc. Roy. Soc.*, **133**, 565 (1931).

the base of the bubble as at the vertex

$$T/R_{bb} + T/R_{ab} = 2T/R_a - E^2/8\pi, \tag{3}$$

where E is the electrostatic field at the vertex, the field at the base being zero. Now from geometry

$$R_{bb} = b, \qquad R_{ab} = a^2/b, \qquad R_a = b^2/a. \tag{4}$$

Substitution of Eqs. (4) and (2) in Eq. (3) gives

$$E^2 = 8\pi\mu^{-5/3}(2\mu^3 - \mu - 1)T/r_0, \tag{5}$$

where

$$\mu = a/b. \tag{6}$$

From the electrostatic theory of a conducting ellipsoid in a uniform field

$$E/E_0 = [e^3/(1 - e^2)]\{\ln [(1 + e)/(1 - e)]^{1/2} - e\}^{-1}. \tag{7}$$

Now

$$e^2 = 1 - b^2/a^2 = 1 - \mu^{-2}. \tag{8}$$

Accordingly, E/E_0 can be evaluated as a function of μ and can be denoted by $f(\mu)$. By means of this E can be eliminated from Eq. (5):

$$E_0(r_0/T)^{1/2} = [8\pi(2\mu^3 - \mu - 1)]^{1/2}/\mu^{5/6}f(\mu). \tag{9}$$

The right-hand member of this equation has been plotted as a function of μ in Fig. 1. It is seen that for values of

$$E_0(r_0/T)^{1/2} < 1.78 \tag{10}$$

there is a value of μ for which Eq. (9) is satisfied, and there is an equilibrium configuration of the bubble. But when 1.78 is exceeded, instability sets in, and were it not for other factors the bubble would certainly explode. Thus the value of the constant in Eq. (1) is

$$c = 1.78 \text{ e.s.u.} = 534 \text{ in } v/\text{cm}. \tag{11}$$

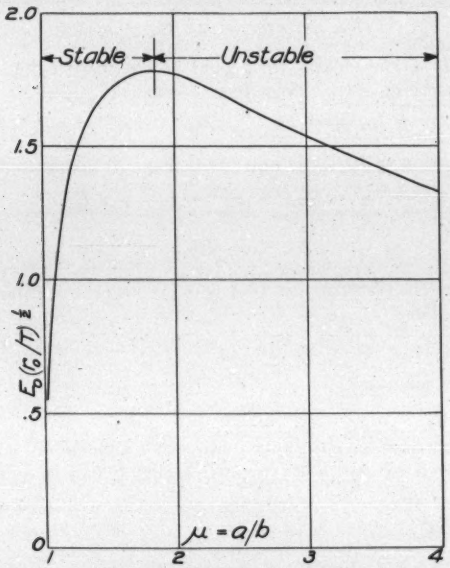
This is reasonably close to the experimental value [5]

$$E(r_0/T)^{1/2} = 1.49 \text{ e.s.u.} = 447 \text{ in } v/\text{cm}. \tag{12}$$

in view of the approximation which has been necessary.

The value of μ at which instability sets in is 1.85. The

FIG. 1.



Equilibrium of a bubble in an electric field.

relative increase in height for any value of μ is

$$\eta = a/r_0 - 1 = \mu^{2/3} - 1 \qquad (13)$$

by Eq. (2). Thus the critical relative increase is

$$\eta_c = (1.85)^{2/3} - 1 = 0.507. \qquad (14)$$

For a 0.67 cm. radius bubble Macky's maximum elongation plotted in his Fig. 1 [4] is 0.28 cm., giving $\eta = 0.28/0.67 = 0.42$. The associated value of μ is 1.69 which corresponds to a field so near the critical that closer agreement between η_{obs} and η_c could not be expected. For a 0.488 cm. bubble the extreme plotted point lies at $\eta = 0.43$, except for one on a dotted extension of the curve, the significance of which is

obscure. (Measurement of the 5200 v cm.$^{-1}$ bubble of Macky's Plate II gave $\mu = 1.75$ but the position of the base of the bubble is too uncertain to attach any weight to this value.) This experimental evidence thus tends to confirm the present theory.

As equality of pressure existed in the soap bubbles experimented with, this fact must underlie any comprehensive theory. Rayleigh's formula for the disruption of a charged spherical droplet is obtained when the right-hand member of Eq. (3) is set equal to zero. Since the left member is the general internal pressure, this would imply that there is no internal pressure originating from surface tension. But it is clear that there is. In fact, the total curvature of the surface at the equator of the ellipsoid is comparable with that at the vertex even beyond the threshold of instability. In applying Rayleigh's formula, Zeleny found it necessary to use an empirical value for the critical μ, whereas the complete equation as used here yields a theoretical value. Thus, the present analysis is an advance beyond the approximation which Rayleigh's formula gives.

These considerations apply only to cases when the electric field is applied so slowly that essentially static conditions obtain. The photographs show definitely that instability sets in by disequilibrium at the vertex. But in the case of the falling drops both theory and comparison with bubble behavior show these simple considerations cannot be directly applied, at least without further justification, even though the value of the disruptive field is strangely consistent with that found for bubbles.

The droplets are undoubtedly in oscillation as they pass through the field region. When they break away from the dropper, vibrations are set up. Besides, the rather sudden application of field attendant upon their emergence into the parallel plate condenser starts oscillations.

Consider the latter cause. The period of oscillation of a water drop in its fundamental mode is [6]

$$\tau = r_0^{3/2}/3.87.$$

For the larger droplets used by Macky [5] $r_0 = 0.25$, whence

[6] H. Lamb, "Hydrodynamics," 5th Edition, 1930, § 275.

$\tau = 0.032$ sec. Assuming that the figure portraying the apparatus, Macky's Fig. 1, is drawn to scale, the distance from dropper to top condenser plate is 17 cm. Using this to calculate the drop speed on entering the field, and taking the distance over which the field changes from zero to full value to be 2 cm., the diameter of the entrance tube, we find the time of application of the field to be 0.011 sec., which is but a fraction of the natural period. The conditions thus approach an impulsive application of the field for which the maximum distortion is twice that which would be caused by an infinitely slow application of the field.

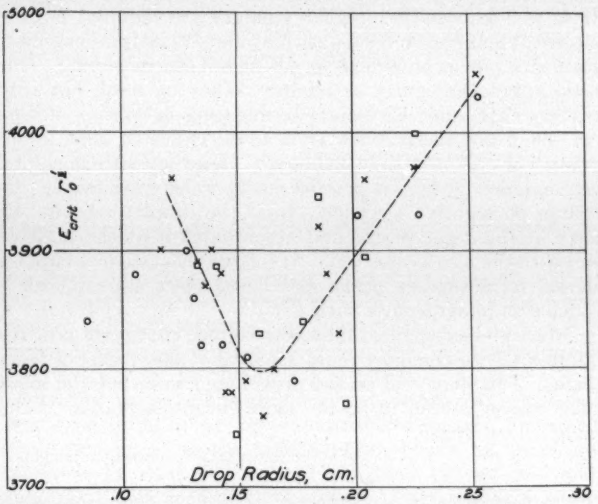
On the experimental side there is the evident tendency to detach a comparatively large globule, at least from the positive end of the droplet, compared to the sharp peaking in the bubbles. This cannot have been due to having the fluid in bulk rather than in a mere shell, but is to be ascribed to the motion already present in the drop as the critical condition was traversed.

On the basis of the theory and the bubble experiments, drops whose lengths are three or more times their equatorial diameter are no longer on the threshold of instability as Zeleny's calculation implies but are already well along in the process of disruption.

The numerical agreement of $E_c r_0^{1/2}$ between the bubble and droplet experiments remains unexplained. There is, however, a small systematic variation in this product with drop diameter, which may be a result of the joint influence of the two causes of oscillations. Fig. 2 is a plot of $E r_0^{1/2}$ against r_0. The crosses are for the vertical field case (Table I),[5] the circles for the horizontal field (Table II), and the squares for the vertical field at reduced pressure (Table IV).

When a bubble has reached the unstable condition, the electric force tends to point the vertex of the bubble more and more as the distortion proceeds. At a certain configuration we can only suppose that the pointing becomes extreme enough and the field is enhanced enough for an electrical discharge to occur. When this happens, the distorting field is suddenly wiped out and surface tension must again draw the bubble back toward the hemispherical form. Only as the surface charges once more by conduction through the film can

FIG. 2.



Systematic variation of $E_{crit}\, r_0^{1/2}$ with r_0. Plotted from Macky's Tables I (crosses). II (circles), and IV (squares).

instability be reëstablished and another cycle started. This explanation of the observed oscillations finds support in the difference between positive and negative bubbles, as appears below, but further confirmation by measurements of the current passed, the oscillation frequency, and the electrical resistance of unstable bubbles is desirable. It may or may not apply to the intermittency observed by Zeleny in supported drops where the bulk liquid furnishes an electrical conductivity which is far greater than that of a thin film.

It is readily seen that inertia carries the oscillating bubble slightly beyond the configuration at which discharge takes place, thus accentuating the pointing. Surface tension cannot be as effective in drawing back the sharply pointed peak, where the force relative to the mass of liquid and its velocity is smaller, as it is in retracting the rest of the bubble. The sharper the peak before the electrical strain is relieved, the

more liquid will be ejected from the bubble in each oscillation. It is well known that higher voltages are required for the electrical breakdown from positive points than from negative. Ratios of potential as high as 1.8 have been recorded. This ratio approaches unity as blunter points are used, but only reaches that value for points terminating in spheres of 0.02 cm. diameter or greater. It is to be expected that liquid points behave in the same manner. That they do is shown by (*E*) and also by (*F*), as pointed out by Zeleny, where the discharge of negative electricity from the droplet relieves the force at the negative end and accentuates it at the positive end with the result observed. It follows that positive bubbles should form sharper peaks and hence eject more liquid, a conclusion in agreement with (*G*).

Macky [5] has ascribed the lower voltage rupture of positive bubbles (*B*) to the more rapid removal of film material from them. This may well be the proximate cause, but the more basic reason is seen to be the more violent oscillation at the same field strength.

ON THE OSCILLATORY MOTION OF A LOADED HELICAL SPRING.

BY

SEIICHI HIGUCHI, Ph.D., Assistant Professor;
TAMAKI TOMITA AND MASATOSHI KASHIWAGI,

Faculty of Engineering, Tohoku Imperial University, Sendai, Japan.

§ 1. INTRODUCTION.

Recently the oscillatory motion of a loaded helical spring was studied by one of the present writers in the hope of obtaining accurate values of the internal viscosity of a solid, owing mainly to the fact that a test piece constructed by a helical spring may be assumed as being one fulfilling approximately the condition of the elimination of clamped effects.[1]

In the course of the studies above mentioned, we at first met some ambiguous points in regard to a somewhat essential problem relating to the motion of the helical spring, which ought to be used, of course, throughout the writers' main treatment with respect to the study of the viscosity of the solid.

The so-called ambiguous points above mentioned are:

1. In practice, the equation of an axial motion of a helical spring is constructed as the type of the harmonic equation [2] under the assumption that the spring is an elastic rod independent of the pitch of its winding. However, speaking generally, the construction of the equation above mentioned must be one which ought to be inquired about and ascertained experimentally in regard to its sphere of application.

2. As is usually known, an equivalent mass of the spring used widely in the equation of motion of a spring having the type of the simple harmonic motion, was taken as $1/3m$ under

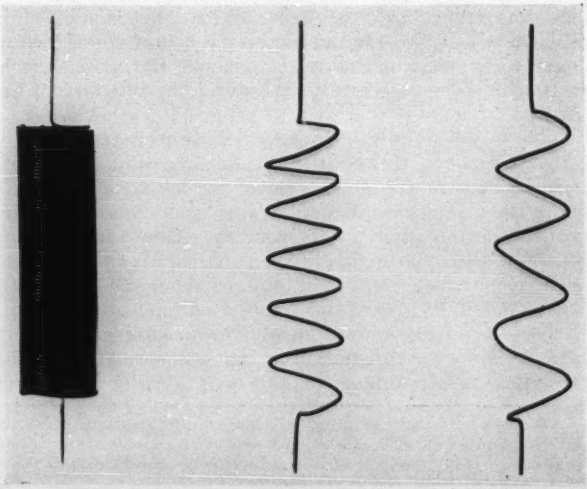
[1] It seems to us that the inequality of the values of the coefficient of the internal viscosity of the solid measured by several authors even on the standpoint of the same fluid-like theory of viscosity, may have an essential relation to the somewhat neglect of the effect of the clamping of the vibrating test piece.

[2] Strictly speaking, for the motion of a helical spring, we have to solve the two simultaneous equations of motion. See, for example, L. R. Wilberforce: "On the Vibration of a Loaded Spiral Spring," *Phil. Mag.*, **38** (1894).

the assumption that it is concentrated at the center of the attached load, provided that $M > m$ where M and m denote respectively the mass of an attached load and that of the spring. On this point, though there are some discussions and conclusions given by the well-known Rayleigh's and Steuding's works,[3] yet, it seems to the writers that, in a word, their conclusions still omit some important points. Specially, the result given in the work of the latter seems to the writers to be an error.

The problem of the so-called ambiguous points above mentioned is indeed a simple one but it seems to us to be very important not only to our studies as already stated but also from the standpoint of the general technical application. This is the reason why, in our reports concerning the viscosity of the solid, besides solving it first of all, we have treated it minutely.

FIG. 1.



Steel, no. 1. Steel, no. 2. Steel, no. 3.

[3] H. Steuding: Messung Mechanischer Schwingungen (1928), S. 359.

Here, the four sets of helical springs made of annealed steel (see Fig. 1) and rolled brass and two kinds of load, which are similar in shape and each of which has a series of different weight, were chosen as the test pieces, and the experiments to measure the natural frequency and the damping factors of the system corresponding to the several springs and attached loads were carried on under the same pressure, 1/200 mm. in Hg, and at almost the same temperature.

On the other hand, based on the results of the above mentioned experiments together with that given in the writers' calculation, the so-called previous questions mentioned above were inquired into closely and, at the same time, some discussion and conclusions were obtained.

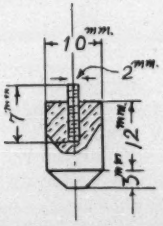
§ 2. EXPERIMENTAL PART.

Helical springs used here are as follows:

TABLE 1.
Dimensions, Mass of Spring etc.

Material.	Radius of helix, cm.	Length of wire, cm.	Radius of wire, cm.	Density.	Simple rigidity c.g.s.
Steel, no. 1	0.639	316.402	0.0281	7.80	$8.036 \cdot 10^{11}$
Steel, no. 2	0.567	21.336	0.0282	"	"
Steel, no. 3	0.565	14.266	0.0283	"	"
Brass	0.569	211.093	0.0346	8.50	$3.912 \cdot 10^{11}$

The loads chosen here are constructed as shown in the next figure [4] and by changing the materials, we have the following sets of weight, viz.,



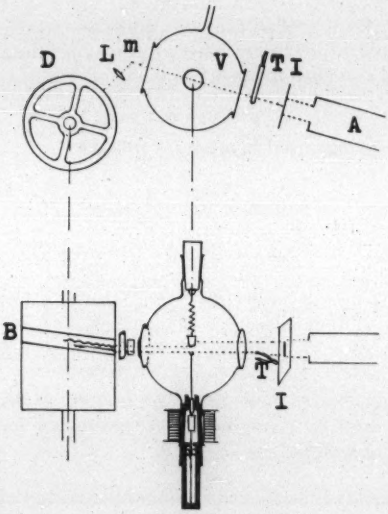
[4] The dimensions shown in the figure correspond to those of the small load mentioned below.

M(small) in gr.; 0.025, 1.01, 1.50, 3.03, 4.26, 4.82, 6.21, 7.79, 9.24, 12.05.

M(large) in gr.; 2.74, 5.92, 9.24, 10.41, 12.74, 15.95, 19.51, 24.77.

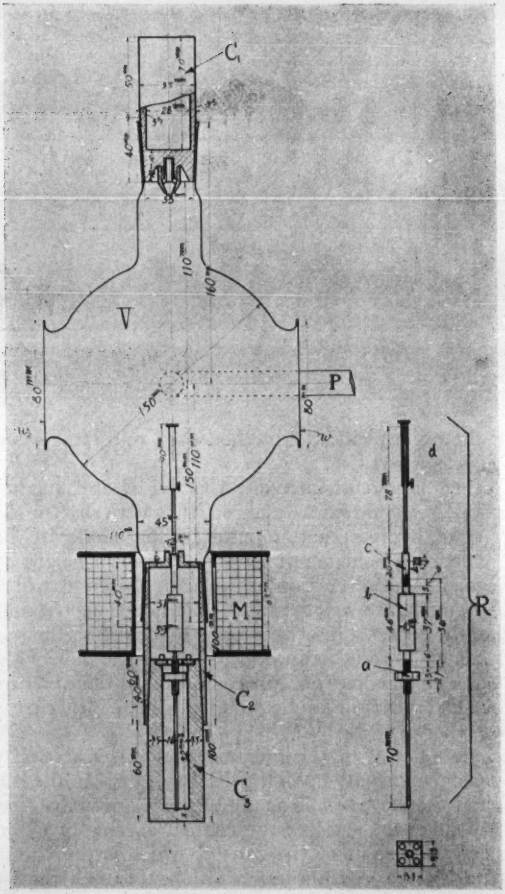
The main parts of the apparatus used in our experiment are shown by Fig. 2 and Fig. 3.

FIG. 2.



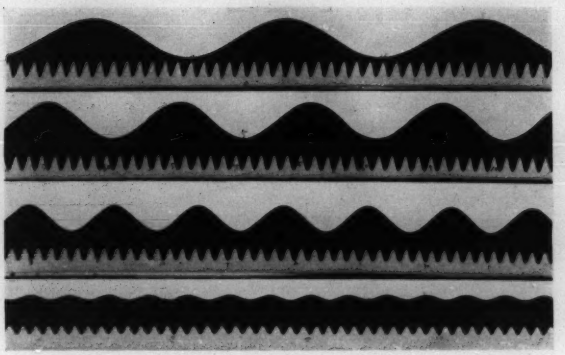
A part of the ray, after passing through I, the space between the lower part of the hanging load and the upper part of the arm of the tuning fork, the mirror m and the cylindrical lens L, strikes finally on the bromide paper covering D. The features of the oscillation of the load and of the tuning fork are photographed on the paper above mentioned. Figure 4 is a part of the photograph obtained by the experiment just stated.

FIG. 3.



(*V*) Glass flask having two plate windows, w_1, w_2, and three mouths. (C_1) Brass cock having in one end a mechanism to clamp the wire of the spring. (C_3) Cock bound by the coil (*M*) and having a hole in one end with square section. (*R*) Rod, of brass, constructed of the following several parts: (*a*) A part having an outer boundary with square section and a screw in contact with the rod. By rotating (C_3), this part travels up and down along the rod vertically. (*b*) A part made of Permalloy, which becomes a magnet at the instant the electric current passes through the coil (*M*). (*c*) A part with square section. (*d*) A cap having a disc on its upper end by which the load may be supported horizontally. (*A*) Arc lamp. (*I*) Screen having a narrow vertical window. (*T*) Tuning fork. (*m*) Plane mirror. (*L*) Cylindrical lens. (*D*) Rotating drum covered with bromide paper (*B*).

FIG. 4.



In the experiment, the method used to set the system in motion is as follows:

Closing the circuit including the coil M and magnetizing the rod (b), we can remove the rod, that is to say the disc (d) without further external impulse in the moderate height. The moderate height here mentioned means a height a little above the balanced position of the load, chosen at will. The so-called moderate height above mentioned is obtained by rotating the cock C_3, in other words, by changing the position of (a) along the rod, because (a) contacts the axis of the rod with screw and moreover is in contact with the inner hole of the cock C_3 by the square boundaries having a little clearance between them.

As the amount of the range of the rod permitted to be removed depends only upon the distance between the position of (a) and the narrow hole at the end of C_3, we have, for convenience sake, the scale at the contact surface of C_2 and C_3 indicating the height of the cap (d) which is prepared from the data of the equilibrium positions of the loads. The reason why the upper narrow hole of C_2 and (c) of the rod are both constructed with square side is, with the help of (a), to hold the rod in a vertical position.

In our experiment, the degree of vacuum in V is always

maintained at 1/200 mm. in Hg by using an oil rotary and Langmuir's pump in series connection. But, in actual fact, the constant pressure above mentioned means mainly carrying on our experiment under constant conditions, because in the present experiment, the existence of air is ascertained to be independent of the frequency of the vibrating system, within sufficient order of accuracy in our experiment.

Now, the frequencies corresponding to the several terminal loads are obtained as shown in the next tables.

TABLE 2.

The Values of M/m and Frequencies.[5]

M.	$(M+m')/m$.		Frequency observed.		Frequency calculated.	
	Steel no. 1.	Brass.	Steel no. 1.	Brass.	Steel no. 1.	Brass.
0.025	0.0077		7.874		7.833	
1.01	0.1678		6.849		6.775	
1.50	0.2494	0.2285	6.410	8.999	6.358	8.960
3.03	0.4986	0.4540	5.465	7.759	5.390	7.707
4.26	0.6992	0.6356	4.926	7.030	4.891	6.991
4.82	0.7917		4.739		4.694	
6.21	1.0179		4.329		4.291	
7.79	1.2764	1.1574	3.984	5.605	3.943	5.679
9.24	1.5141	1.3728	3.717	5.209	3.685	5.316
2.05	1.9730	1.7883	3.333	4.625	3.302	4.773

[5] m' is the mass of a part of the wire of the spring measured from the lower end of the helix to the free end which is inserted into the load.

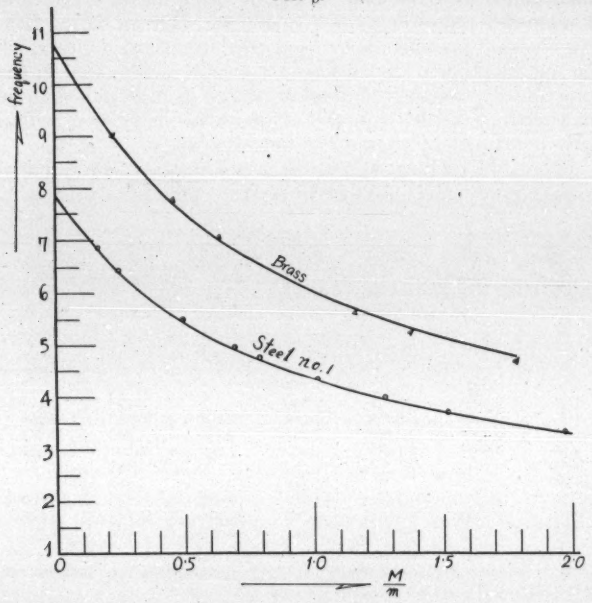
TABLE 2.

The Values of M/m and Frequencies (continued).

M.	$(M+m')/m$.		Frequency observed.		Frequency calculated.	
	Steel no. 2.	Steel no. 3.	Steel no. 2.	Steel no. 3.	Steel no. 2.	Steel no. 3.
2.74	7.173	10.709	31.073	38.191	30.958	38.208
9.24	22.767	33.660	17.777	21.626	17.570	21.779
5.95	38.856	58.064	13.685	16.666	13.462	16.670
9.51	47.381	70.806	12.448	15.178	12.200	15.055
4.77	60.012	89.685	11.120	13.495	10.848	13.383

Plotting the above, we have obtained Fig. 5 and Fig. 6. In these figures the smooth curves and the marks denote the respective values of the frequencies calculated and observed.

FIG. 5.



The calculated values above mentioned are illustrated fully later.

§ 3. THEORETICAL PART.

The equation of motion of a helical spring having the load at its free end expressed in the type of the simple harmonic is

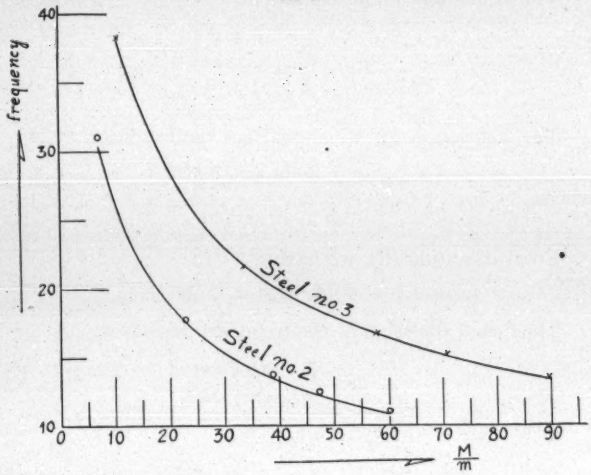
$$\left(M + \frac{1}{k}m\right)\ddot{\xi} = -\left(\frac{\pi n r^4}{2la^2}\right)\xi = -c^2\xi, \qquad (1)^6$$

where

ξ Elongation of the spring,
l, r Total length and radius of the wire,
M, m Mass of the attached load and that of the spring,

[6] Of course, the calculations shown below are not new for us.

FIG. 6.



n Coefficient of rigidity,

k Constant depending on the ratio of M/m, etc. which is mentioned already in § 1,

a Radius of the helix.

Putting $M + (1/k)m = M_1$, $c^2/M_1 = q^2$, we have the expression for the frequency corresponding to the load attached as

$$f = q/2\pi = 1/2\pi \left[\frac{\pi n r^4}{2la^2 \left(M + \frac{m}{k} \right)} \right]^{\frac{1}{2}}. \qquad (2)$$

On the other hand, we have the equation of motion of the spring constructed under the assumption that the system in question may be assumed as a loaded elastic bar as

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{c^2 L^2}{m} \frac{\partial^2 \xi}{\partial x^2} = q_1^2 \frac{\partial^2 \xi}{\partial x^2}, \qquad (3)$$

where L denotes the length of the helix.

The boundary conditions are

$$\left.\begin{aligned} &\text{(i)} && \xi = 0 && \text{at } x = 0, \\ &\text{(ii)} && M\ddot{\xi}_t = -c^2 L \frac{\partial \xi}{\partial x} && \text{at } x = L. \end{aligned}\right\}$$

The solution of (3) which satisfies the condition (i) is

$$\xi = \sum A \sin \eta x \cdot e^{i\sigma t}, \tag{4}$$

where

$$\eta^2 = \sigma^2/{q_1}^2. \tag{4_1}$$

From condition (ii), we have

$$\tan(\eta L) = c^2(\eta L)/M\sigma^2 = m/M(1/\eta L). \tag{4_2}$$

Then the expression of the frequency becomes

$$f = \sigma/(2\pi) = \frac{1}{2\pi}\frac{\eta L}{\sqrt{m}}\left[\frac{\pi n r^4}{2la^2}\right]^{\frac{1}{2}}. \tag{5}$$

Equating (2) and (5), we have

$$k = 1/\{(\eta L)^{-1} - M/m\}. \tag{6}$$

From (4_2), we will first obtain the values of ηL corresponding to M/m and introducing these values into (6), we can easily determine the values of k. In Table 3, Fig. 7, the relation

TABLE 3.

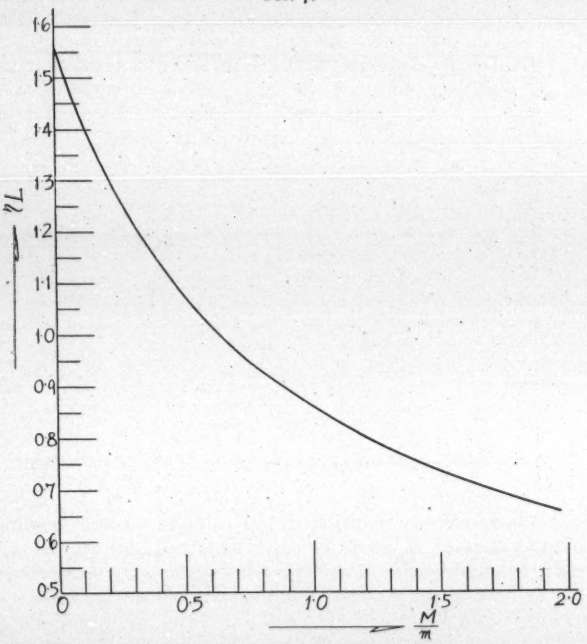
The Relation between M/m, ζL and k.

M/m.	ηL.	k.	M/m.	ηL.	k.
0	1.5708	2.4680	1.2764	0.7846	2.8746
0.0077	1.5588	2.4760	1.5141	0.7332	2.8912
0.1678	1.3483	2.6159	1.9730	0.6570	2.9120
0.2494	1.2651	2.6639	4.0000	0.4801	2.9513
0.4986	1.0777	2.7595	5.0000	0.4338	2.9601
0.6992	0.9733	2.8053	20.0000	0.2218	2.9977
0.7917	0.9341	2.8212	∞	0.0000	3.0000
1.0179	0.8547	2.8488			

between ηL and M/n are shown, and similarly in Table 3 and Fig. 8 we have the relation between k and M/m.

Here, it is very interesting to compare the above mentioned results with those shown in Rayleigh's and Steuding's

FIG. 7.

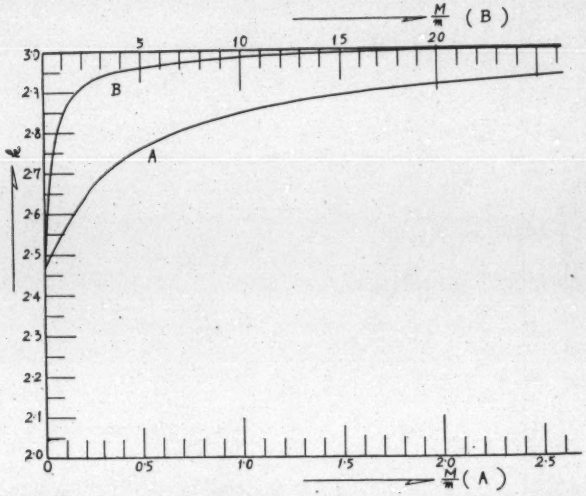


works:—The larger the value of M/m becomes, the nearer the value of k comes to 3, and for the limiting case when $M/m \to \infty$, we have easily evaluated the limiting value of k as 3, eliminating M/n between (4) and (6), and so on.

Thus, it may be clear naturally, that the results and discussion given in Steuding's work,[7] namely that the value of $k = 3$ above mentioned mean only a mean value, require somewhat essential correction.

[7] Since we have had no opportunity to see the paper published by A. H. Krylow which is the original paper of the discussion quoted in Steuding's work, there is a doubt about the discussion mentioned above whether it is quoted correctly. However, in our opinions, it seems to be correct to say, at least, that the distinct explanation of the problem in question is not a matter of common knowledge.

FIG. 8.



Now, as already mentioned, the value of k corresponding to any values of M/m can be easily obtained and we can calculate the frequencies of the vibrating systems by using equation (2). The results thus obtained are shown in Table 2_1, 2_2, Fig. 5 and Fig. 6.

The close coincidence of the results calculated and observed shown in these figures seems to us to indicate that (1) the equation of motion of a common helical spring having the type of the harmonic one, constructed by the method already mentioned, is correct independent of the pitch of the spring, and (2) the law which governs the equivalent mass of the system in question is accounted for from both the theoretical and the experimental standpoints of view.

§ 4. SUMMARY.

The main results obtained here are summarized as follows:

1. Usual conventional equation of motion of a common helical spring may be used without considering the pitch of its winding.

2. The fractions of the equivalent mass of the spring corresponding to the several loads attached are clearly deduced, showing as its special case, that the value of $1/3m$ is a mere limiting value of the case when $M/m \to \infty$. Accordingly, it may be worthy of notice that the meaning of $1/3m$ given in Steuding's work requires some correction.

In conclusion, the writers wish to express their hearty thanks to the Nippon Gakujutu Sinkokwai for its financial aid in accomplishing this work. Our sincere thanks are due also to Mr. I. Iinuma for his zealous help in the present investigation.

Sorbitol and Mannitol.—(*Chemical Industries*, *38*, 57.) In the group of new chemical products displayed at the recent Chemical Exposition were two members of the glycerin family, mannitol and sorbitol. Sorbitol until this time had been a laboratory curiosity, quoted at $400 a pound, the other, mannitol, has been available only as an importation.

Every development has a good reason for its beginning and one of the earliest of the so-called modern explosives was made by nitrating mannitol. This nitro-mannitol has some very desirable properties as an industrial explosive but its cost in the past has been prohibitive. Atlas Powder Company's attempts to produce a synthetic mannitol at reasonable cost has resulted in the two related hexahydric alcohols being synthesized from corn sugar and they are now available in ton quantities.

Sorbitol like mannitol is found in nature in minute quantities in mountain ash berries, apples, pears, cherries and other plants. Now available in commercial quantities sorbitol promises to be a most important intermediate in the preparation of Vitamin C (ascorbic acid). Naturally occurring Vitamin C has been obtained from red cabbage at the rate of one gram of vitamin per ton of vegetable. Prepared synthetically from sorbitol, ascorbic acid has been brought down to a small fraction of its former cost. Other applications for sorbitol are to be found in the textile industry as a conditioning agent; in the tanneries for softening and finishing skins; in printers' rollers as a life prolonger.

Mannitol is synthetic mannite, which was the active ingredient of the manna of the Old Testament, and for long centuries was a favorite confection-laxative of the peoples of the Mediterranean countries. An interesting use is in the test for boron. For this purpose a considerable quantity is consumed annually by city water supply departments.

C.

CHLADNI FIGURES ON SQUARE PLATES.*

BY

ROBERT C. COLWELL, Ph.D.,

Professor of Physics, West Virginia University.

It has been shown before that the amplitude of the vibration of a square plate is given approximately by the equation.[1]

$$W = \left\{ \begin{array}{l} (\cos k_m a - \cosh k_m a)(\cos k_m x + \cosh k_m x) \\ + (\sin k_m a + \sinh k_m a)(\sin k_m x + \sinh k_m x) \end{array} \right\} \times \left\{ \begin{array}{l} (\cos k_n a - \cosh k_n a)(\cos k_n y + \cosh k_n y) \\ + (\sin k_n a + \sinh k_n a)(\sin k_n y + \sinh k_n y) \end{array} \right\} \quad (1)$$

and that for higher vibrations (large values of m and n) equation (1) reduces to

$$W = A \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{a} \quad (2)$$

for points on the plate not too near the edges; along the edges, equation (1) must be used. The nodal lines are then found in the usual way by solving the approximate equation

$$W = A \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{a} + B \cos \frac{n\pi x}{a} \cos \frac{m\pi y}{a} = 0. \quad (3)$$

Equation (3) is the solution for a supposititious membrane with free edges.

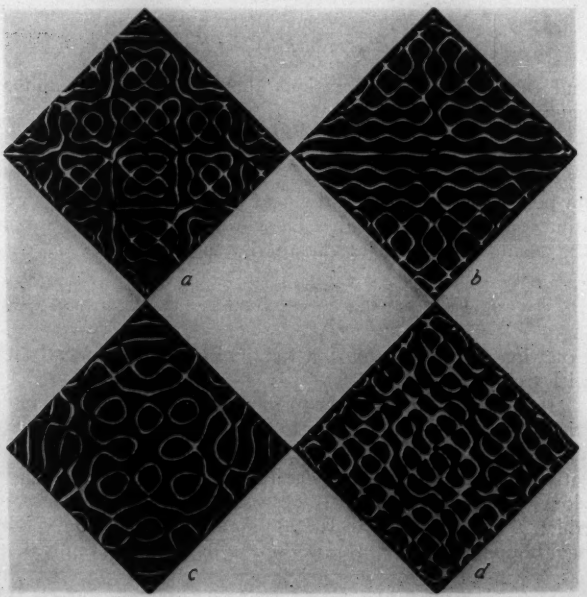
If the membrane is in the form of an isosceles right triangle with the equal sides of length a lying along the coördinate axes with the origin at the corner, the conditions to be satisfied by the free edges are that $\partial W/\partial x = 0$, $\partial W/\partial y = 0$ along the equal sides and $\partial W/\partial n = 0$ along the hypotenuse, in which n is the normal to the hypotenuse in the plane of the membrane. The condition $\partial W/\partial n = 0$ is however equivalent

* Publication Number 101, Division of Industrial Arts and Sciences, West Virginia University.

[1] Colwell, JOURN. FRANKLIN INST., 214, No. 2, Aug., 1932, p. 202.

to the first two conditions since n_x the component of n in the x direction is $n \cos 45°$ (more generally $n \cos \alpha$) and $n_y = n \sin 45°$ (more generally $n \sin \alpha$). Therefore, the solution for the triangle is given by equation (2); and equation (3) will give the sand figures for the triangle. The nodal lines may be plotted for a triangle and two such triangles placed together along the hypotenuse to form a square. However, the two hypotenuses may be fitted together in two ways, one of which gives exact symmetry about the diagonal of the square (Fig. 1, *a* and *b*); the other gives what we have called one hundred and eighty degree symmetry. It is obtained by rotating one triangle through 180° and fitting it to the other (Fig. 1, *c* and *d*).

FIG. 1.



The nodal lines in (*a*) and (*b*) are symmetrical about the median diagonal, each half being a mirror image of the other. In (*c*) and (*d*) either half must be rotated 180° in order to fit over the other.

If the membrane is in the form of a rectangle with the origin at the lower left-hand corner and the sides b and a lying along the axis of x and y respectively, then the solution will be (using equation 3)

$$W = A \cos\frac{m\pi x}{b}\cos\frac{n\pi y}{a} + B\cos\frac{n\pi x}{b}\cos\frac{m\pi y}{a} = 0. \quad (4)$$

If in particular $b = a/2$, the rectangle is exactly one-half the square of sides a. Hence it is possible to calculate the nodal lines for each rectangle and to fit two of them together so as to produce a square. The two rectangles may also be fitted together in two ways, one of which produces mirror symmetry about the median line, the other 180° symmetry. Examples of these are drawn in Figs. 2 and 3.

This method of joining together two triangles or two rectangles to form a square is also applicable to two equal semicircles. Since the line along which the two semicircles are joined to form a circle is always satisfied by the equation $\partial W/\partial n = 0$ in which n is the normal to the diameter forming the edge, the two semicircles will give mirror symmetry or 180° symmetry. Figure 4 shows two examples of the latter type in which any diameter whatsoever will divide the circle into two exactly similar parts.

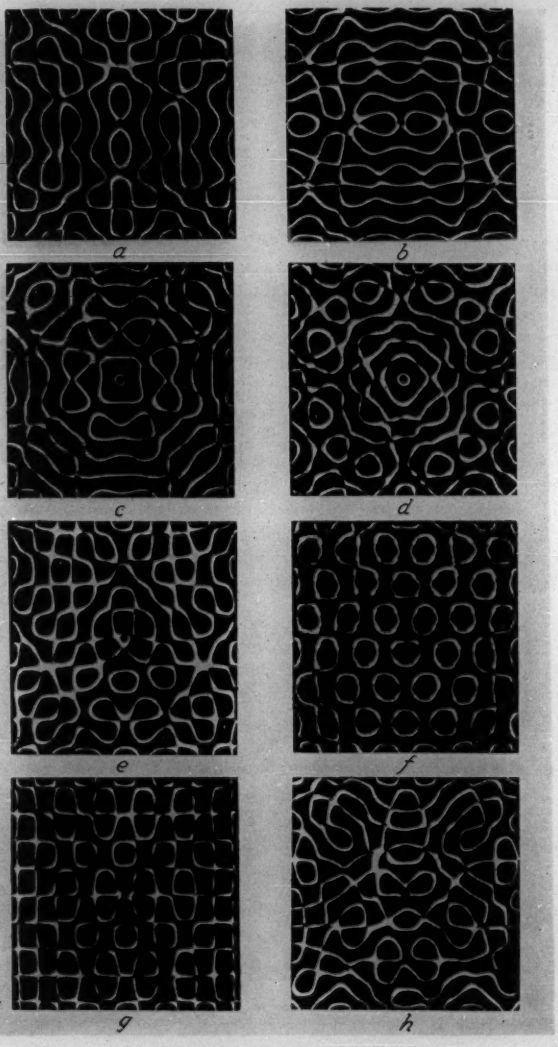
For a rectangle with $b = a/2$, equation (4) takes the form

$$W = A \cos\frac{2m\pi x}{a}\cos\frac{n\pi y}{a} + B\cos\frac{2n\pi x}{a}\cos\frac{m\pi y}{a} = 0. \quad (5)$$

The rate of vibration depends upon the term $m^2/a^2 + n^2/b^2$,[2] so that for any given note this expression must be constant. For the square $a = b$, so that only $m^2 + n^2 = a$ constant need be considered. The simplest case occurs when m and n are interchanged in equation (2) and the two terms added together to form equation (3). There are other values for m and n which are found from the theory of numbers; such combinations are (1, 7) and (5, 5); (1, 8) and (4, 7); (6, 7) and (9, 2); and so forth. The method outlined here of regarding the square as compounded of two rectangles gives an additional

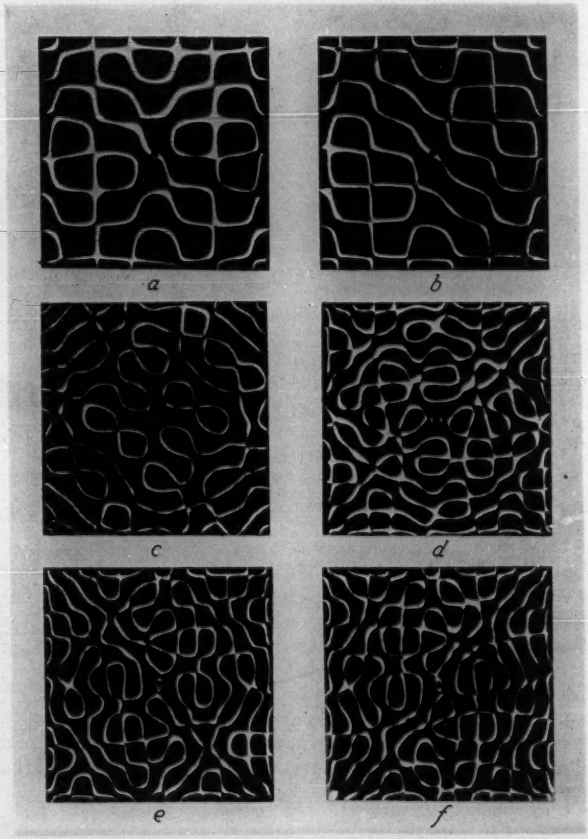
[2] Rayleigh, "Sound," Vol. I, p. 307.

FIG. 2.



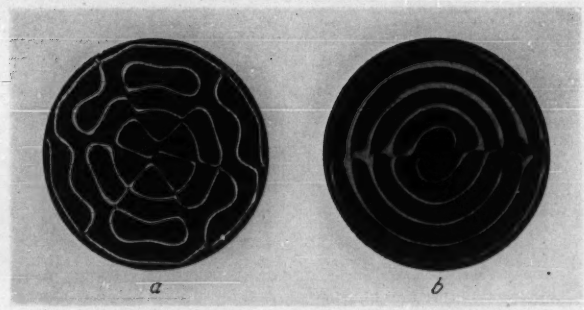
All the sand forms are symmetrical about the median line parallel to the sides of the square, one-half of each figure is the mirror image of the other half.

FIG. 3.



If each square is cut into two equal rectangles by a median line parallel to the sides; one rectangle rotated through 180° will fit over the other.

FIG. 4.



Two circles with 180° symmetry. Any diameter will give two similar semicircles.

series of numbers found from the equation $m^2 + 4n^2 = a$ constant (or $4m^2 + n^2 = a$ constant). A few of these groups obtained by the use of number theory are (2, 2) and (4, 1); (2, 4) and (8, 1); (2, 5) and (10, 1); (2, 6) and (12, 1) etc. The first set (2, 2) and (4, 1) inserted in equation (3) gives

$$W = A \cos\frac{4\pi x}{a}\cos\frac{\pi y}{a} + B \cos\frac{2\pi x}{a}\cos\frac{2\pi y}{a} = 0. \qquad (6)$$

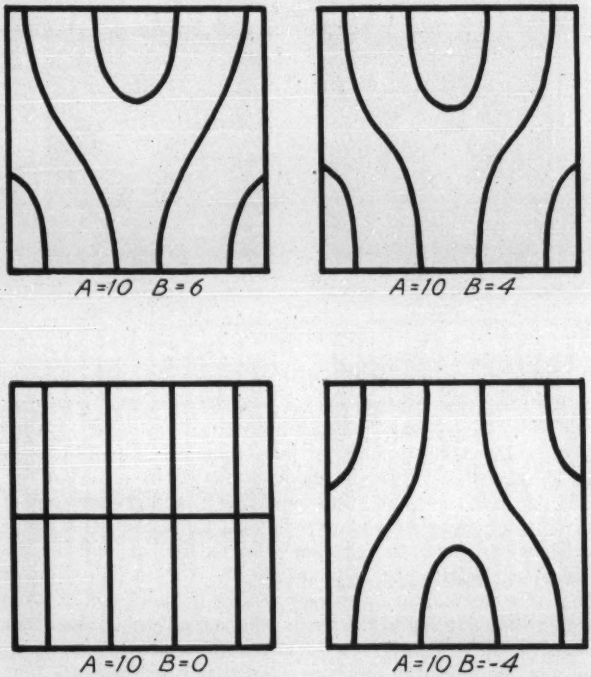
Four diagrams plotted from this equation are shown in Fig. (5). This method gives a new series of Chladni figures which has not been explained before. A photographic reproduction of one of these is shown in Fig. 6*a*.

Since when either A or B is zero, equation (3) reduces to a series of parallel lines equally spaced across the plate, it seemed that a large difference between A and B ought to produce the wavy parallel lines which are sometimes seen in a vibrating plate (Fig. 6*b*). A trial of this method produced the lines shown in Fig. 7.

The theoretical lines in Fig. 8 have been plotted from equation (3). The dotted lines in the figure at the lower right hand corner are plotted from equation (1); they represent almost exactly the sand lines found by experiment.

The physicist working with these plates is confronted with

FIG. 5.

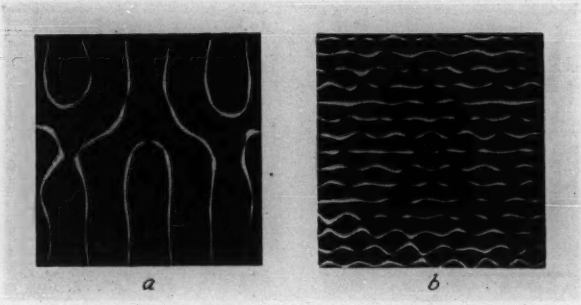


These nodal lines are possible figures on a square plate, although they are formed from the combination of two rectangles. For each figure $m_1 = 4$, $n_1 = 1$, $m_2 = 2$, $n_2 = 2$. They are plotted from equation 6.

two problems neither of which is easily solved—first, given the figure find the mathematical equation; second, given the equation, produce the figure. A very great aid to the solution of both problems is found in a statement made by Rayleigh.[3] He remarked that in whatever ratio A and B are taken, the

[3] Rayleigh, "Sound," Vol. I, p. 314.

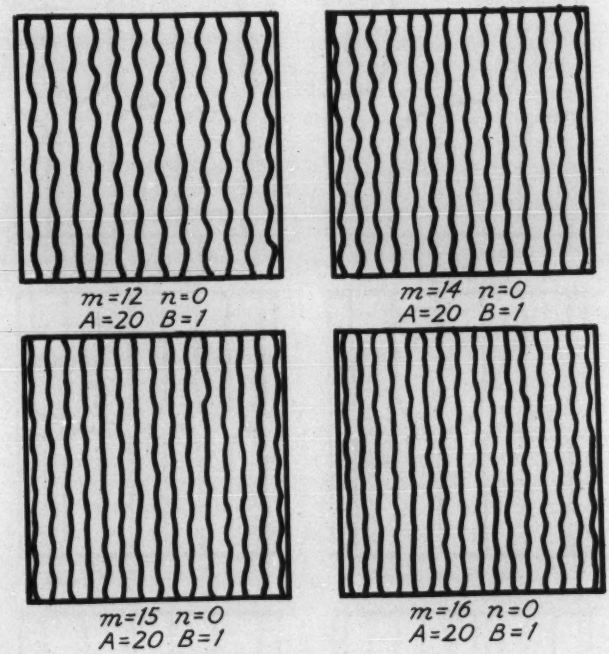
FIG. 6.



The lines in (*a*) represent a new type of nodal curve found from the rectangle. There are fifteen wavy lines across (*b*).

nodal lines always pass through the points of intersection of the nodal lines of the two cases $A = 0, B = 0$. An important generalization of this is that any given note will always produce the same number of lines upon the membrane (or plate) regardless of the ratio of A and B in equation (3). When $B = 0$, the single term (equation 2) will give a series of straight lines across the plate parallel to the sides. If B is not equal to zero, the straight lines are twisted around on the plate, but their number is unchanged. Thus in Fig. 5 one of the plates has four straight lines parallel to the y axis and one straight line parallel to the x axis; the nodal lines on the other plates of Fig. 5 always cut the edges of the plates exactly as the straight lines did. In this particular case there are four points of nodal intersection with the x axis and one point with the y axis. The same thing is shown in Fig. 8 for different values of m and n. In this case m gives the number of points of intersection with the x axis and n the number of intersection on the y axis. This method is useful in many cases but is not universally applicable on account of the fact that some lines curl up completely to form closed curves near the edges of the plate: It is then difficult to determine the points along the axis. However, in trying to find the mathematical equation of any given figure, it is advisable to count

FIG. 7.

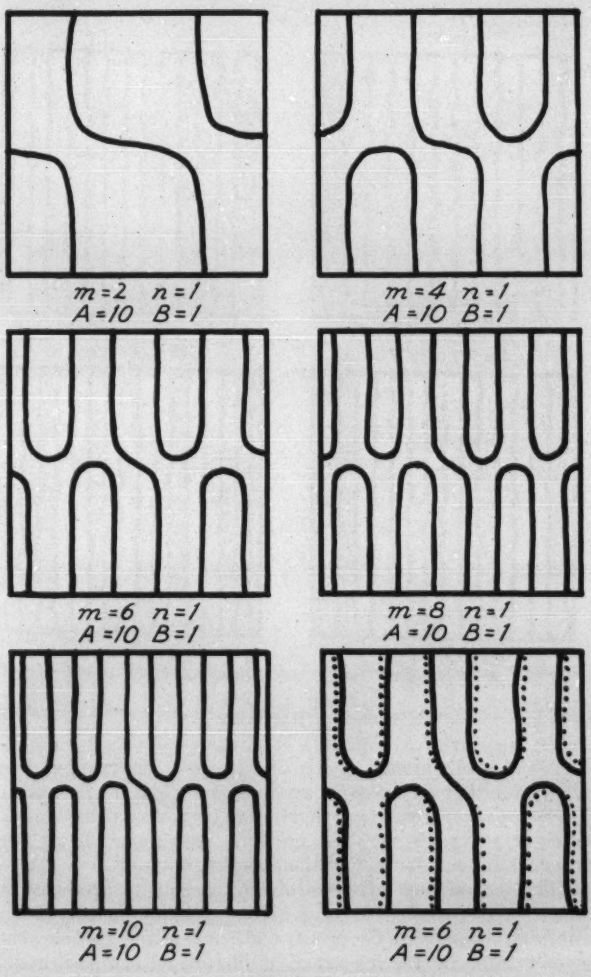


These figures show the wavy lines for different values of m and n. The ratio of A to B is constant and equal to 20 in equation 3.

the nodal points along the axes of x any y—these two numbers will give either an exact or approximate value for m and n. The values for A and B are then found by trial in equation (3), different values being used until the theoretical figure approaches the appearance of the experimental one.

The second part of the problem, namely to produce the figure corresponding to any given equation is solved by clamping the plate at the proper nodal points and applying the correct note in the electrical oscillator. Both the nodal

FIG. 8.



The nodal lines are plotted from equation 3 and show the increasing complexity as m and n increase in value. The dotted curves in the lower right-hand square are calculated from equation 1.

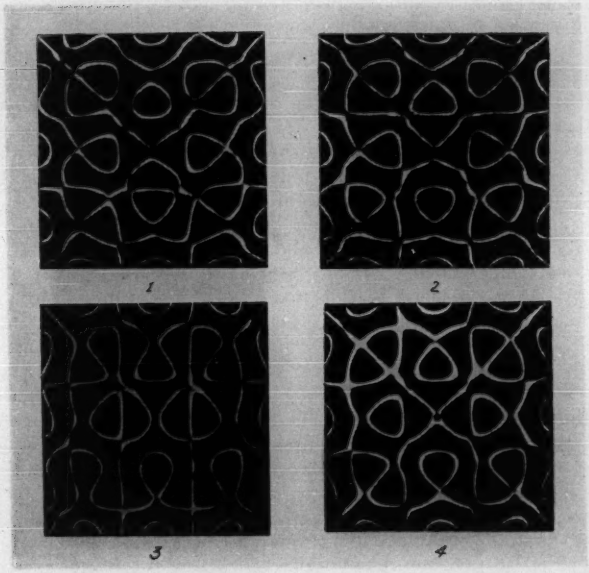
points and the number of vibrations are found approximately by calculation from equation (3) or equation (1).

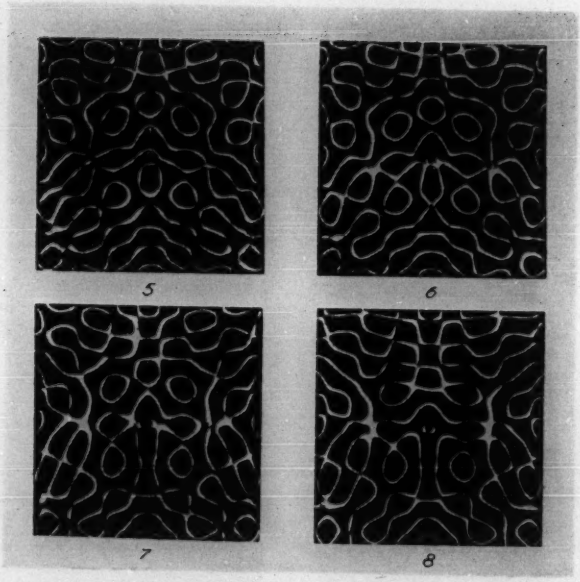
In the former method of vibrating a Chladni plate with a violin bow, the operator had to satisfy four constants at the same instant (that is A, B, m and n). If he were fortunate enough to hit on m and n, he still had a great deal of difficulty in producing A and B in the proper proportions. With the electric oscillator, the operator always has m and n exactly correct and he can hold this note indefinitely; but the determination of A and B is still a matter of trial. The attempt has been made with some success to fix A and B by using different kinds of supports for the plates. Usually the circular hole of the Chladni plate was resting on a conical support; this was varied with a spherical support; a support with four edges and so on. Small weights set in the plate at different places also tend to produce a definite pressure on the diaphragm of the oscillator and so fix the ratio of A to B.

The statement has often been made that the Chladni figures must be symmetrical about the center and (or) the four corners of the plate. It has been shown here that this is only true when the whole plate is taken as the vibrating unit and A made equal to B. It is also possible to divide the plate into rectangles or triangles and combine these to form a square. In this case both rectangles (or both triangles) must have the same value of m and n, but the values of A and B may be different in each segment. Thus it is possible to find a great number of curves which at first glance seem to be lacking in symmetry but which are nevertheless solutions for equation (3) provided the segments are properly chosen. These figures were not often found with the violin bow because the custom of clamping the plate at the middle and bowing at the edge tended to give figures symmetrical about the center of the square. This is an unnecessary limitation.

In Appendix I are shown several figures with rectangular and triangular symmetry as well as the more usual symmetrical type. The first eight figures show two separate families in each of which m and n are the same but A and B vary.

APPENDIX I.

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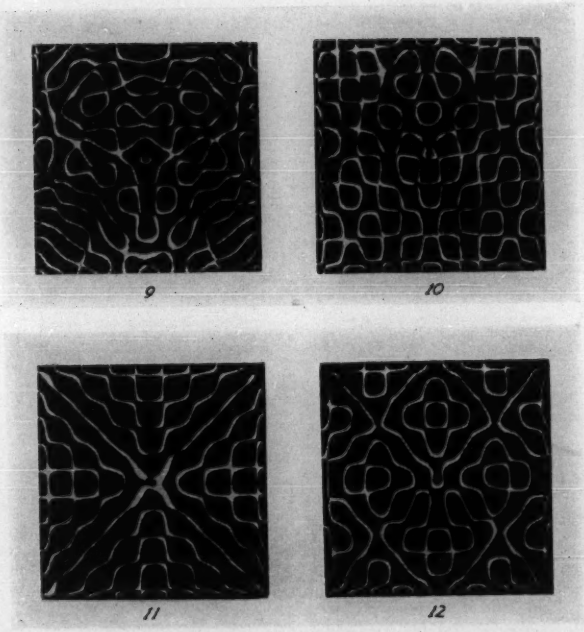
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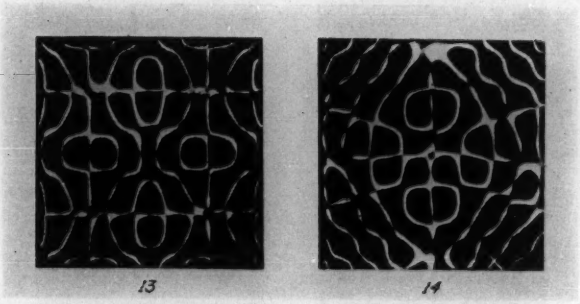

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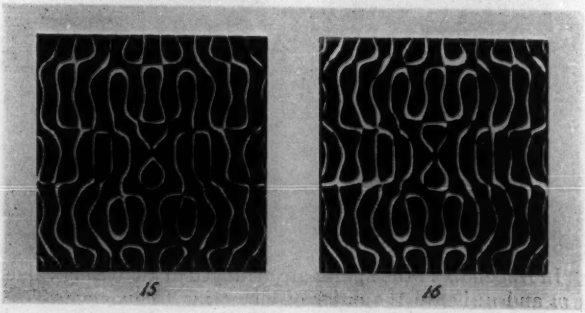
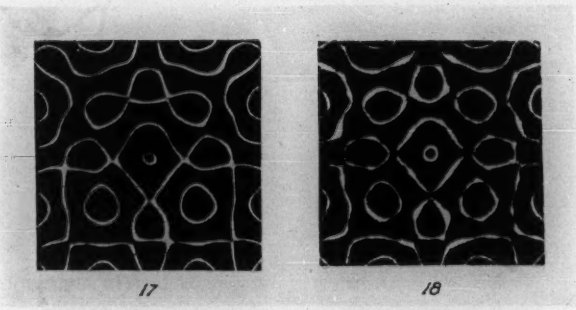


15 16



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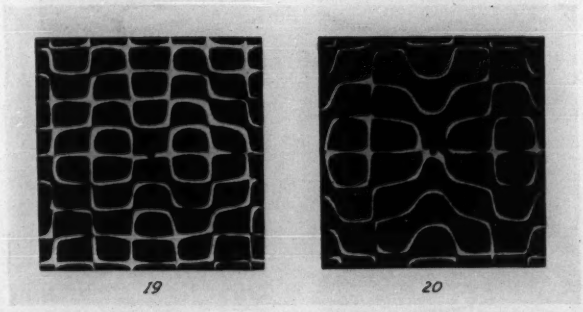
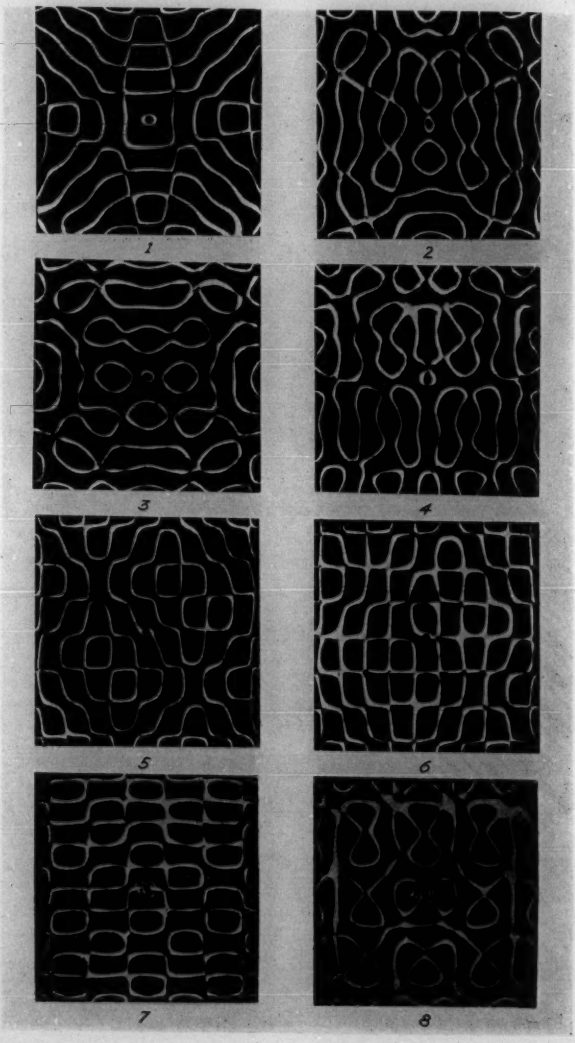
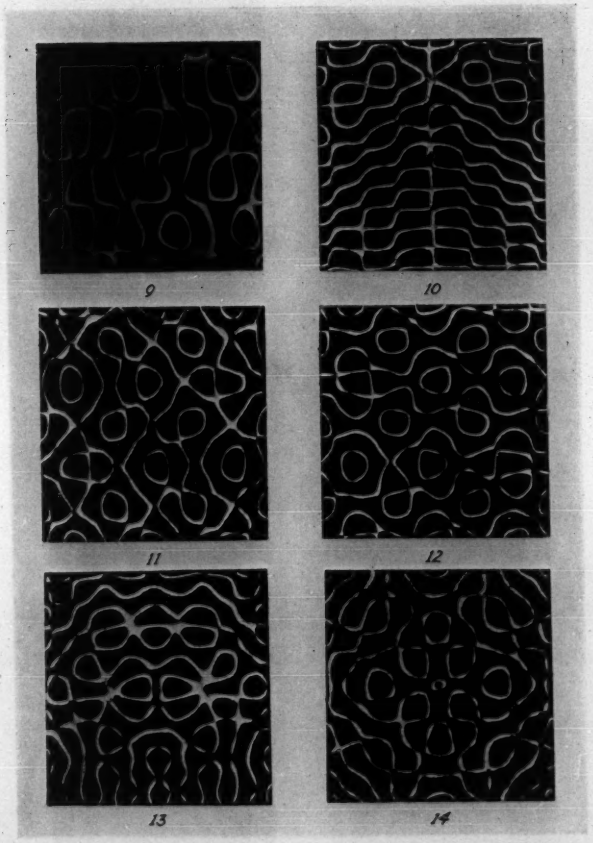


19 20

In Appendix II the figures are arranged in increasing values of m and n; i.e. in the order of increasing frequency. They illustrate the fact that with higher notes, the number of nodal lines is greater.

APPENDIX II.

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A STUDY OF THE QUALITY, THE DESIGN AND THE ECONOMY OF CONCRETE.

BY

INGE LYSE,

Research Associate Professor of Engineering Materials, Lehigh University, Bethlehem, Pennsylvania.

(*Continued from last issue.*)

d. Strengths of Concrete.

Recent investigations have gone far to establish the relations between the ingredients in the concrete mix and the strengths of the resulting product. It has already been pointed out that since the cement is the only chemically active ingredient in the concrete and reacts with the mixing water, the relation of cement to water must necessarily be the major determining factor for the qualities of the concrete. In the following sections a discussion is given of the different strength properties of concrete. Special emphasis has been given to the compressive strength because this strength not only to a large extent reflects the other strengths of the concrete, but also indicates qualities such as modulus of elasticity, permeability, durability, fireproofing, etc. Thus the compressive strength becomes the controlling factor for most of the qualities desired of the concrete, and a detailed discussion is given of the laws which govern this strength.

e. Compressive Strength.

Since the concrete is made up of a binding part, the cement paste, and a filler, the aggregate, the quality of the concrete is determined primarily by the quality of the paste. This was realized already in 1897 by R. Feret (4). He found that the strength of cement mortars was determined by the amount of cement per unit of voids (water + air) plus cement, or in other words, by the amount of cement per unit of paste in the mortars. His experimental evaluation of the relationship

was:

$$f_c' = K\left(\frac{c}{1 - V_s}\right)^2, \qquad (1)$$

where f_c' is compressive strength, K is a constant depending upon the materials and conditions of tests, c is amount of cement, and V_s is absolute volume of sand. Assuming that the same law would hold for concrete, the equation becomes:

$$f_c' = K\left(\frac{c}{1 - V_a}\right)^2, \qquad (2)$$

where V_a is the absolute volume of the total aggregates, fine and coarse.

Feret's law states that the strength of mortars and concrete is determined by the concentration of cement particles in the cementing paste. This is a very logical law and all later developments have more or less unconsciously supported this law. In 1914 M. O. Withey (5) published results which showed a straight line relation between the strength of concrete and its cement-void ratio, another expression for the concentration of cement particles in the paste. Professor Withey's relation may be expressed by the following equation:

$$f_c' = A + B \cdot \frac{c}{v}, \qquad (3)$$

where A and B are constants depending upon the materials and conditions of tests, and v is amount of voids (water + air).

The development of the water-cement ratio seemed for a while to discredit the Feret's law. While also the water-cement ratio is an expression of the concentration of the cement in the paste, an overemphasis was placed on the water content, ignoring to a large extent the cement. Professor Abrams' water-cement ratio relation (1), first published in 1918, is given by the equation:

$$f_c' = \frac{A}{B^{w/c}},$$

where w is the water content. The vast amount of research which followed the development of the water-cement ratio

theory placed the water content ahead of the actual quality of the cementing paste, and the close agreement between Feret's law and the water-cement ratio relation was lost sight of. At the University of Illinois, A. N. Talbot and F. E. Richart (6) published results in 1923 which showed a very good agreement with Feret's law. Their equation for average conditions was:

$$f_c' = 32{,}000 \left(\frac{c}{c+v} \right)^{2.5}. \qquad (5)$$

Except for the exponent 2.5, instead of 2.0, this equation is identical with Feret's equation.

The importance of the quality of the cementing paste regained much ground by the publication of the "Basic Principles of Concrete Making" by F. R. McMillan (3), and Professor Slater's (7) presentation of the harmony between the water-cement ratio and the cement-space ratio relations. A further emphasis on the quality of the paste was given by R. L. Bertin (8) in 1930. He found that Abrams' water-cement ratio curve became very nearly a straight line when the strength of the concrete was plotted against the specific gravity of the paste. His equation was:

$$f_c' = A \cdot g_p - B, \qquad (6)$$

where g_p is the specific gravity of the paste. A more simple expression for the quality of the paste in the concrete was presented by the author in 1931 * (9). It was shown that both the water-cement ratio and the cement-space ratio curves became very closely straight lines when the quality of the

* As early as 1925 the Swiss Committee on Gussbeton in its report, Part 9 by E. Stadelmann, proposed the strength equation:

$$f_c' = A \frac{c}{w} - B,$$

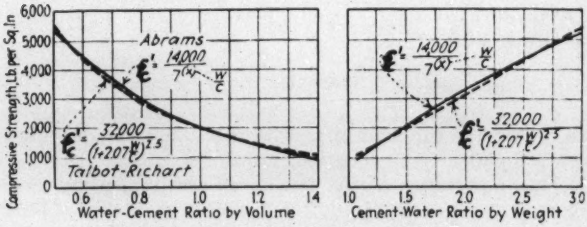
and J. Bolomey (10) has proposed the equation:

$$f_c' = K \left(\frac{c}{w} - 0.50 \right).$$

The first one of these formulas is identical to the one developed independently by the author. The formula by Bolomey is more restricted insofar as it only applies for certain cases. It may be said that Bolomey's formula represents the special case of the author's formula when A is one-half of B.

paste was expressed by the cement-water ratio. Fig. 9 shows how the relationships for the water-cement ratio and the cement-space ratio become nearly straight lines when ex-

FIG. 9.



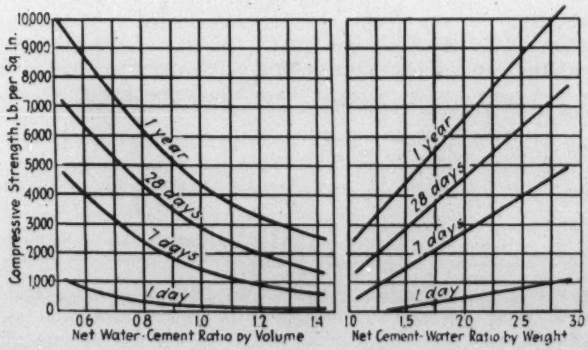
Abrams' and Talbot-Richart strength curves when plotted on the basis of cement-water ratio.

pressed in terms of the cement-water ratio. The cement-water ratio relation is given by the equation:

$$f_c' = A + B \cdot \frac{c}{w}. \qquad (7)$$

This straight line relation holds fairly well within a range which covers nearly all practical concrete mixes. Fig. 10

FIG. 10.



Strength of concrete at different ages on basis of water-cement ratio and cement-water ratio.

shows how each age of test has its own straight line strength-cement-water-ratio relationship (11). Very lean, as well as very rich mixes generally do not follow the straight line relation, but the relationship between the quality of the paste and the strength of the concrete is not dependent upon a straight line function. The general law for the strength of the concrete may therefore be expressed as follows: *For cement contents above a minimum required for giving placeability and binding strength of the concrete, the strength increases regularly with the increase in the concentration of cement particles in the water of the paste.*

All the relationships given above have the same basic conception, they all recognize the quality of the paste as the quality giving ingredient of the concrete. However, the quality of the paste is expressed in different terms, the cement-space ratio, the cement-voids ratio, the water-cement ratio, the specific gravity of the paste, and the cement-water ratio. For properly produced concrete the amount of air voids is negligible so that the voids are equal to the water content for all practical purpose. The water content is measured directly for all concrete mixes, while the voids have to be determined by calculations from indirect measurements. Consequently the water content is by far the more convenient to use in actual concrete construction. The quality of the cementing paste is therefore expressed most simply by the cement-water ratio.

Experiments have shown, as pointed out in Part II, that for a given type and gradation of aggregate, the water requirement for a given placeability of the concrete is nearly the same for lean and rich mixes alike. Making use of this approximately constant water content for different mixes, the strength equation becomes:

$$f_c' = A + (B/w) \cdot c = A + K \cdot c. \qquad (8)$$

This constant water content theory forms the basis for the "simplified method of design," which is presented in Part II, and also gives the foundation for the rational study of the economy of concrete mixes, presented in Part III.

f. Tensile and Transverse Strengths.

Concrete is weak in tension as compared with compression, and in general design the use of the tensile properties of the concrete is not permitted. However, a knowledge of the tensile strength of the concrete is essential for the prevention of cracks from shrinkage and temperature changes by means of reinforcement. Furthermore, the transverse or the tensile strength in flexure is important for plain concrete structures, such as dams, retaining walls, etc. The tensile and transverse strengths of concrete have not been studied as fully as has the compressive strength, but the data available indicate that the same factors which determine the compressive strength also, to a large extent, determine the tensile and transverse strengths. However, attention should be called to the fact that the characteristics of the aggregates have a greater effect upon the tensile and transverse strengths than upon the compressive strength. This is due to the greater effect of the bond between paste and aggregate on the tensile and transverse strengths. However, for a given aggregate the strengths in tension and flexure follow much the same laws as the compressive strength. The concentration of cement in the paste of the concrete determines the strength qualities. Fig. 11 presents a typical illustration of the general relationship between compressive, tensile, and transverse strengths (12). It is noted that both the tensile and the flexural strengths increased regularly with the increase in the compressive strength of the concrete. Roughly, the ratio between tensile and compressive strengths is one-tenth, and between transverse and compressive strengths one-fifth to one-seventh.

g. Shear Strength.

Professor Mörsch (13) has shown the following theoretical relation between shear, tensile, and compressive strengths:

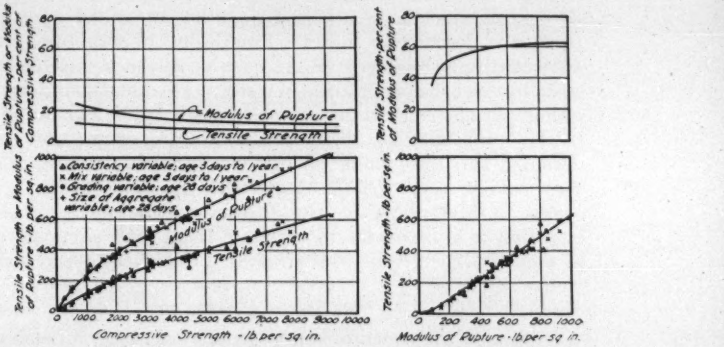
$$s = \sqrt{f_c' \cdot t}, \qquad (9)$$

where s, f_c' and t are shear, compressive, and tensile strengths, respectively. Since the tensile strength may roughly be taken as one-tenth the compressive strength, the relation becomes:

$$s = \sqrt{f_c' \cdot \frac{1}{10} f_c'} = 0.316 f_c'.$$

It is very difficult to study strength in pure shear experimentally, and the results obtained from investigational studies yield very great differences in strength values. In general, the experimental data have indicated a slightly greater strength than the 31.6 per cent. of the compressive strength given by Mörsch's theory. However, the shear value is of practically no importance to the reinforced concrete designer. In ordinary beams and slabs the maximum shear stress can never exceed the maximum tensile stress and since the tensile strength is only about one-third the shear strength, the failure

FIG. 11.


Tensile Strength or Modulus of Rupture - Per cent of Compressive Strength
Modulus of Rupture
Tensile Strength
Tensile Strength - per cent of Modulus of Rupture
Consistency variable; age 3 days to 1 year
Mix variable; age 3 days to 1 year
Grading variable; age 28 days
Size of Aggregate variable; age 28 days
Modulus of Rupture
Tensile Strength
Compressive Strength - lb per sq. in.
Tensile Strength - lb per sq. in.
Modulus of Rupture - lb per sq. in.

Relation between compressive, tensile, and transverse strength of concrete. From Gonnerman and Shuman.

will be produced by tension. When the term shear is used in connection with diagonal tension in beams and slabs it is used in the wrong sense, such usage arises from the fact that the diagonal tension in the section below the neutral axis is equal in magnitude to the horizontal shear and acts in a direction 45° with the horizontal plane. Consequently the diagonal tension is computed most readily by means of the formula for the shearing stress.

h. Bond Strength.

The bond between the concrete and the reinforcement is also to a large extent, dependent upon the compressive

strength of the concrete. Both the initial bond and the final bond may be considered directly proportional to the strength of the concrete. Abrams (14) has shown that for pull-out tests the initial bond was on an average between ten and fifteen per cent. of the compressive strength of concrete of ordinary compositions. The final slip was usually between twenty and twenty-five per cent. of the compressive strength.

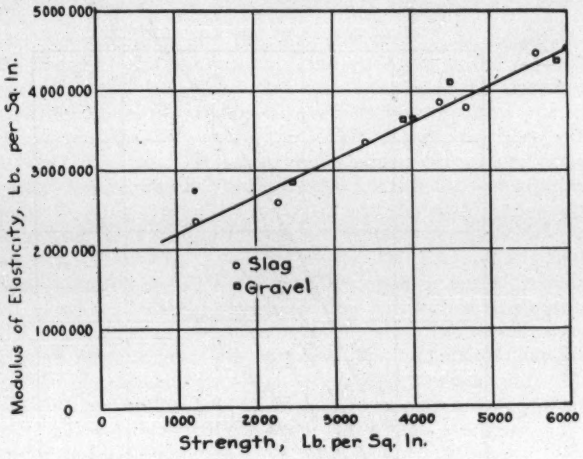
i. Qualities Other Than Strength.

In general, the plain or reinforced concrete structure is exposed to conditions for which qualities other than the strength alone must be taken into consideration. For all out-door structures the weathering qualities are called upon to resist the destructive forces. Practically all structures are subjected to some degree of fire hazard, and many are called upon to resist the penetration of water. Heat and sound insulation are also important in many cases. The elastic and plastic properties of the concrete are important for the designer, and the volume changes are of such a nature that they cannot very well be neglected. In order to have a sound understanding of the factors which control these qualities, it is necessary to correlate them, and particularly to compare them with the strength properties of the concrete.

j. Elastic Properties of Concrete.

Although the concrete is not, strictly speaking, an elastic material, the stress-strain relationship is a measure of its elastic behavior. At very low stresses the strains may be assumed to be proportional to the stresses, however, permanent sets occur at stresses which are quite low because of the effect of plastic flow. The modulus of elasticity is usually based on the initial relationship between stress and strain which is practically a straight line for a stress up to about 25 per cent. of the compressive strength when rapid application of load is used. The relationship between this initial modulus and the strength of the concrete is for ordinary cases approximately a straight line. Fig. 12 shows the straight line relation between modulus of elasticity and strength which was recently obtained in an investigation at Lehigh University on concrete containing gravel and slag as aggregates. The re-

FIG. 12.



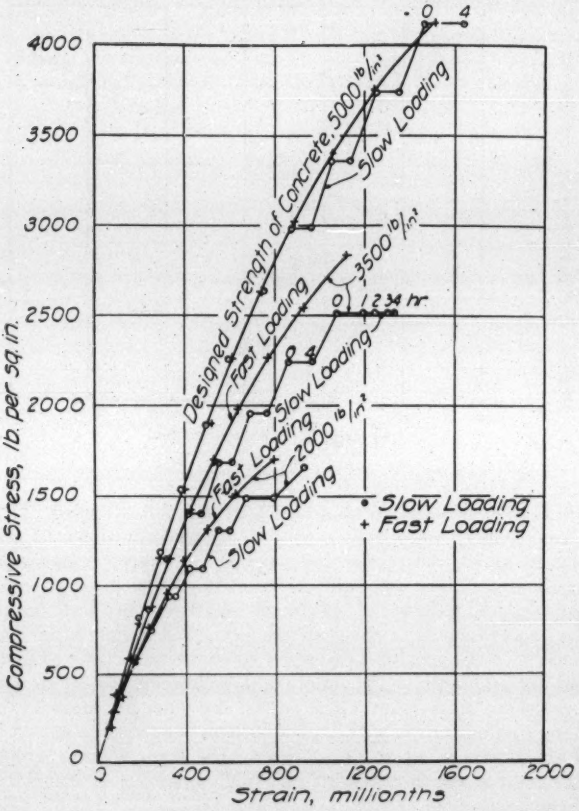
Modulus of elasticity of concrete having gravel and slag as coarse aggregate.

sults were obtained on standard 6 by 12-inch cylinders, with 1/10,000 inch Ames dials recording the deformations. Each plotted point represents the average for three companion specimens. Since one-half of the points in the diagram were obtained on 7 days old specimens and the other half on 28 days specimens, the effect of the age itself seems to be immaterial. All specimens were cured moist until time of test, and tested damp. The important conclusion to be drawn from these tests is that the initial modulus of elasticity is increasing uniformly with the increase in compressive strength of the concrete, or in other words, the same factors which give high strength will also give high modulus of elasticity of the concrete.

k. Plastic Flow.

It is of comparatively recent date that the attention of the engineering profession has been called to the so-called plastic flow of concrete. Experience has shown that when a concrete member is subjected to load for a long time, the

FIG. 13.



Effect of speed of loading and strength of concrete on deformation of plain concrete columns.

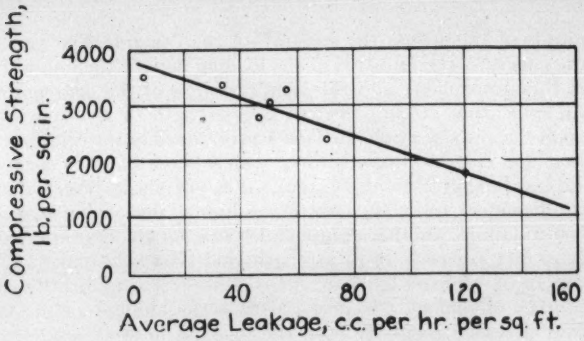
member will continue to deform, although the load remains constant. Experimental studies (15) have now given much information on the problem of plastic flow. It has been found that plastic flow depends upon such factors as the mag-

nitude of the stress, the strength of the concrete, the age of the concrete, the duration of the loading period, the humidity of the atmosphere, and the characteristics of the aggregates. An indication of the effect of the strength of the concrete upon the amount of plastic flow was obtained in the reinforced concrete column investigation which recently was carried out at Lehigh University (16). Fig. 13 shows the stress-strain curves for plain concrete columns, one group loaded fast to failure, another group loaded fast for the first approximate fifty per cent. of its strength, and then held under each increment of load for four hours before the application of another increment. It is noted that the amount of plastic flow increased regularly with the increase in stress and that the weak concrete showed more flow for a given stress than did a stronger concrete. However, when the stress was a certain percentage of the strength, the amount of flow was practically the same for the different grades of concrete. The plastic flow is thus tied in with the compressive strength of the concrete in much the same manner as are most of the other desirable qualities.

l. Permeability.

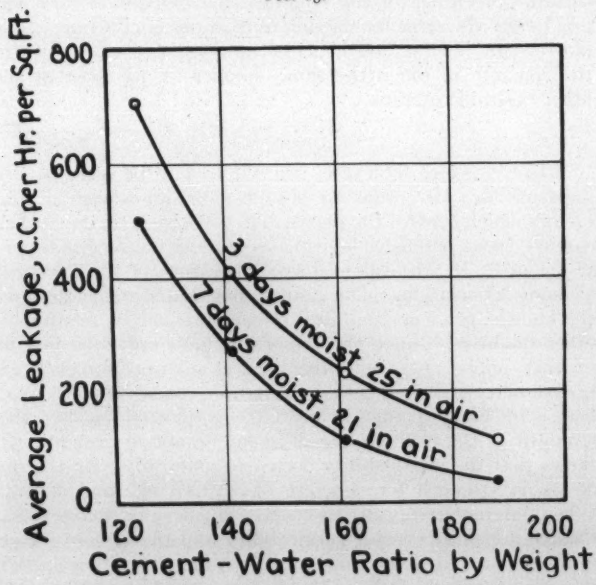
Most concrete structures are required to be more or less impermeable. In order to obtain more knowledge of the factors which control the permeability of concrete, the author carried out an extensive investigation under the general supervision of F. R. McMillan at the laboratory of the Portland Cement Association. The results contributed much towards the knowledge of permeability of concrete and its relation to other qualities. Since the important data are published in another paper (17), only the general relationship between permeability and compressive strength of concrete is presented here. In Fig. 14 the permeability is plotted against the strength of the concrete for different brands of cement. It shows that the permeability decreases uniformly with the increase in strength for concrete of a given age and curing. When different compositions of the cementing paste are used, the relationship between permeability and the cement-water ratio follows much the same trend as does the strength and the cement-water ratio. The leakage of the concrete de-

FIG. 14.



Relation between strength and permeability of concrete made from different cements.

FIG. 15.



Effect of cement-water ratio on permeability of concrete.

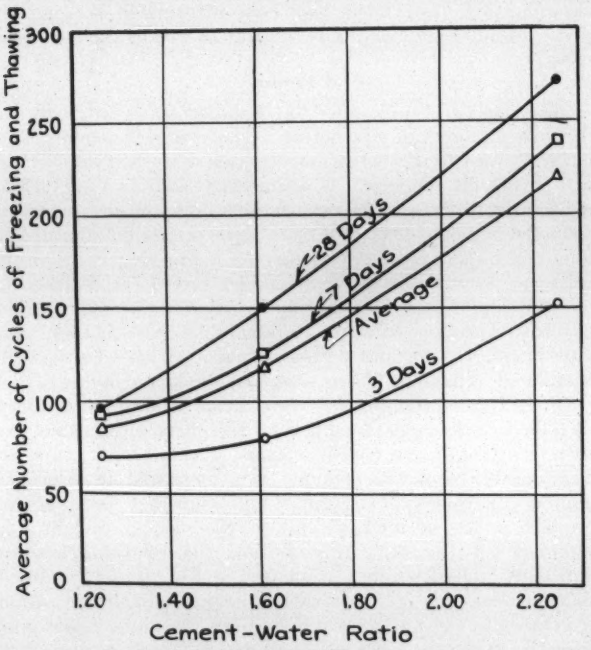
creases regularly with the increase in cement-water ratio as shown in Fig. 15. Thus high concentration of the cement paste produces the most impermeable as well as the strongest concrete.

m. Durability.

Exposed structures must be designed so as to resist the destructive forces of weathering. Impermeable concrete will naturally resist destructive forces much more effectively than will permeable concrete. Consequently there is a very close relationship between the permeability and the durability of concrete, and one of the best tests of the durability is probably found in the permeability experiments. Freezing and thawing tests are, however, a more direct measure of the weathering resisting qualities. Some durability experiments were carried out in connection with the permeability investigation previously referred to, and a greater investigation was recently completed at Lehigh University. In the latter investigation (18), ten different aggregates were included in order to study the effect of the aggregates besides the effect of other factors such as strength and length of moist curing of the concrete. The durability of the concrete was measured in terms of number of cycles of freezings and thawings required to produce a loss of 25 per cent. of the original weight. A summary of the data representing the relation between durability and strength is presented in Fig. 16. It is noted that the durability increases with the increase in concentration of cement in the mixing water in much the same manner as does the strength. The effect of moist curing is also very similar, the age at test representing the length of moist curing, except that the early moist curing was even more effective in increasing durability than strength of the concrete. That the characteristics of the aggregates are important for the durability of the concrete is shown in Fig. 17. For a given aggregate, however, the durability increases in accordance with the same law as does the compressive strength.

The resistance of concrete specimens to the destructive action of sulphate waters follow much the same trend as does the compressive strength (19). Fig. 18 shows that the relation between exposure ratio and water content per unit of

FIG. 16.



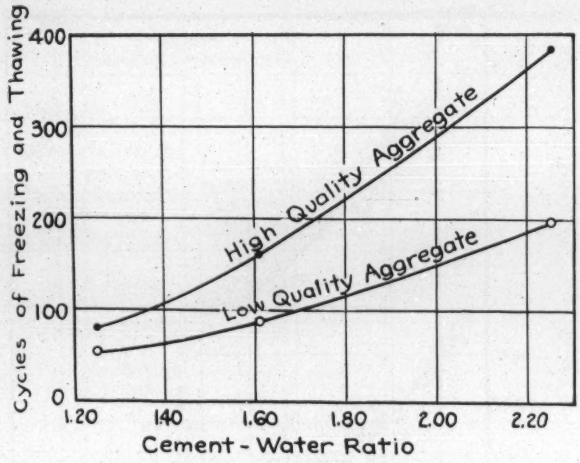
Relation between durability and cement-water ratio of concrete.

cement in the concrete is very much the same as the relation between the compressive strength and water content. The more cement per unit of water the higher is the strength and the higher is the resistance to sulphate waters.

n. Fire Resistance.

Recent investigations at the research laboratory of the Portland Cement Association (20) have revealed some very important facts regarding the fire resistance and heat insulation qualities of concrete at high temperatures. While the

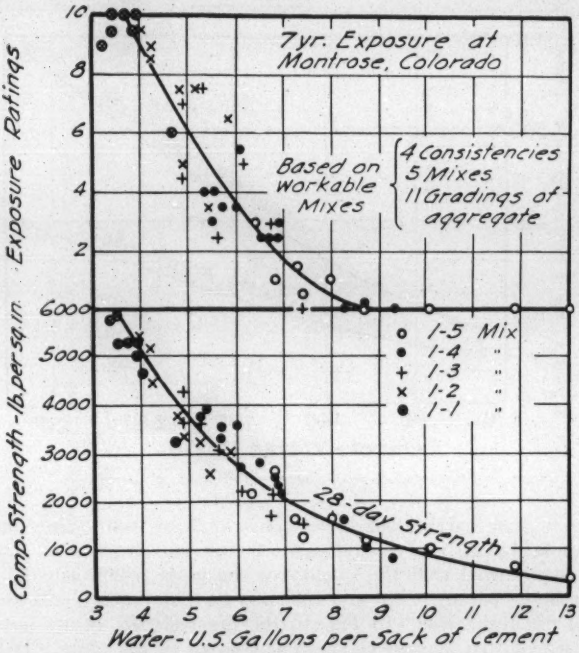
FIG. 17.



Effect of characteristics of aggregate on durability of concrete.

tests were carried out on masonry building units, they are applicable to the general case of concrete structures. The tests showed that the quality of the paste as measured by cement content of the concrete was the outstanding criterion of fire resistance. In Fig. 19 the fire resisting values have been plotted against the cement content of the units. It is noted that the fire resistance increases in direct proportion to the increase in cement content for all the three different aggregates used. This is in perfect harmony with the strength results, indicating that the same factors govern both the fire resistance and the strength. The effect of the characteristics of the aggregates, however, is seen to be much more prominent for fireproofing than for compressive strength. The artificial light-weight aggregate Haydite is particularly well suited for concrete of high fire resistance. Any concrete made from porous or light-weight aggregates would naturally have similar advantages over concrete made from ordinary solid aggregates.

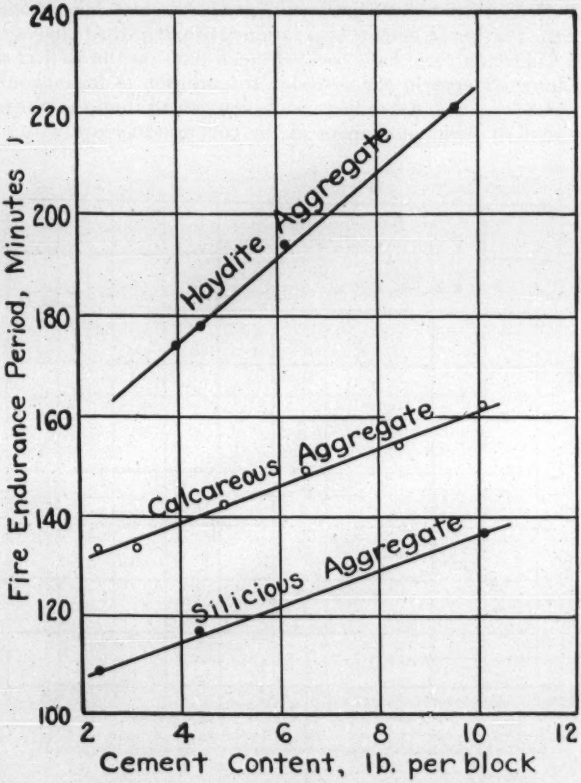
FIG. 18.



Effect of quality of paste of concrete on resistance to sulphate waters. From Wilson and Cleve.

In discussing the different qualities of concrete, the outstanding fact is the controlling influence of the concentration of cement particles in the paste of the concrete. There is a very close relationship between the compressive strength and all the above mentioned qualities of concrete. The compressive strength thus becomes a very useful, in fact almost an essential, test for nearly all desirable qualities of the concrete. High strength indicates high modulus of elasticity, low plastic flow, low leakage, high durability, and high fire resistance. However, there is one quality which unfortunately does not

Fig. 19.



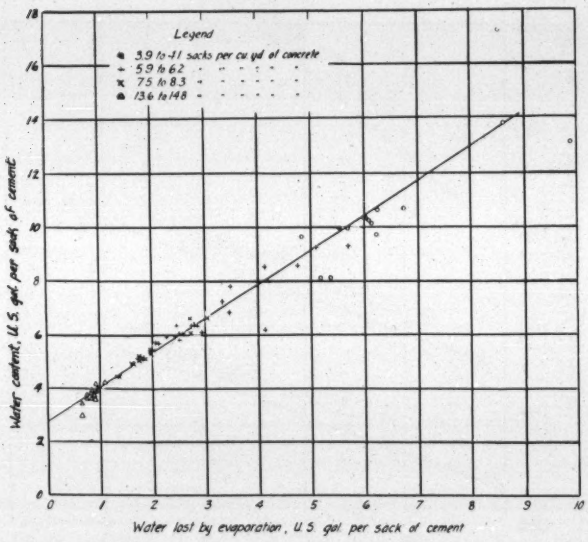
Effect of cement content on fire endurance of walls of concrete masonry units.

follow the same general law; that is, volume changes due to variation in moisture content. The volume changes are generally greatest when the cement concentration is greatest, that is, when all other desirable qualities are present.

o. Volume Changes.

Recent extensive experiments at the research laboratories of the Portland Cement Association (21) and of the University of California (22), have yielded much data on the subject of volume changes in concrete due to variation in its moisture content. The outstanding conclusion gained from these tests is that the volume changes of the concrete increase with an

FIG. 20.

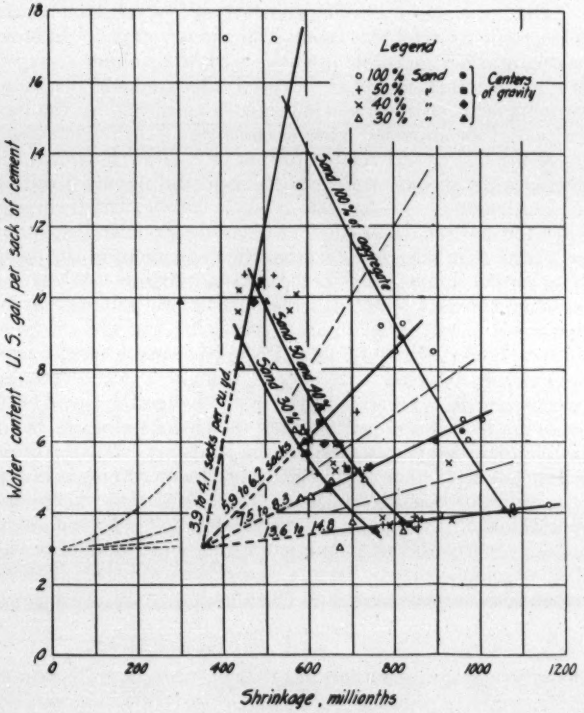


Relation between water in paste and evaporation during dry storage. From Slater.

increase in the cement concentration in the paste, and is greater for a large than for a small amount of paste. Rich mixes and wet mixes are therefore producing more volume changes than lean and stiff mixes. The interrelation (6) between volume change, cement content, and water content, is presented in Figs. 20 and 21 for concrete specimens cured seven days in the moist room at 70° F. and then in the observa-

tion room at a relative humidity of 50 per cent. and a temperature of 70° F. until the age of six months. The average shrinkage due to the loss in weight has been plotted for a

FIG. 21.



Interrelation between water content, cement content and shrinkage of concrete. From Slater.

concrete containing 30, 40, 50 and 100 per cent. sand of the total aggregates. The effect of the amount of sand on the volume change is principally due to the fact that a high sand content requires a larger paste content than does a low

sand content. However, different types of aggregates as well as different types of cements will show differences in the amount of volume changes. Fig. 21 indicates that an initial shrinkage of 0.000350-inch per inch took place, regardless of cement content and paste content.

The principle for making concrete of low volume changes is low paste content, this means lean and dry mixes. In order to secure other desirable qualities with low volume changes, dry mixes must therefore be used which require vigorous tamping or mechanical placing such as vibrating, for the production of a compacted mass of concrete.

A problem closely related to that of volume changes is the temperature expansion due to the heat development during the hardening of the concrete, and the subsequent shrinkage when the concrete cools off. The heat development is greater for a high than for a low concentration of cement in the paste in a similar manner as is the volume change. While for ordinary concrete structures the effect of these temperature changes is mostly of a hypothetical nature, recent very large constructions, such as Boulder Dam, have made certain provisions for reducing this effect. Complete water cooling systems are incorporated in the concrete for the purpose of removing heat developed, so that the entire mass will be at mean annual temperature before the joints are grouted. This will insure no further shrinkage due to temperature drop and thus prevent opening of joints and formation of cracks. Special low heat developing cements are also used as an aid in keeping the expansion at a minimum and reducing the amount of subsequent cooling. Future experiences will show if these precautions are necessary and effective.

(*To be continued.*)

RANDOM FRACTURE OF A BRITTLE SOLID.

BY

C. C. LIENAU,

University of Chicago.

(*Continued.*)

AVERAGE MECHANICS AND GEOMETRY OF FRACTURE STRAIN.*

Granted that the material of the solid is statistically homogeneous and isotropic in all relevant qualities, it is reasonable to suppose that so far as the material itself is concerned it is as likely to yield new surface by fracture at one physical point as at another and that no orientation of the surface is preferred. Assume that such a material composes an ensemble of solids. Through the fracturing action of arbitrary external forces new surface is produced. Failure is supposed to initiate only by parting along a surface of positive tension.[3, 4, 5] The material is perfectly elastic and obeys Hooke's Law up to the point of failure. There is no friction or hysteresis and the temperature of the solid is constant and far from its melting point. All displacements are small. It is then asked what are the necessary average conditions on the displacements and forces at the boundary, and within the solid, if the new surface produced is random in distribution and orientation. Given these conditions it is asked what the expected form of new surface and fragment will be.

To avoid repetition, the term "point" will be understood as the physical point. Continuity is a property of the macroscopic or mean stresses and motions. About these mean values random fluctuations may be discoverable if we can exercise the necessary discrimination within the physical point, and choose to do so. To the relative order of precision defined by the physical point and the mean diameter of the

* The experimental work is understandable without a prior reading of this attempt to anticipate the shape and size probabilities.

solid, the ultimate positive tension stress u, the volume modulus k, shear modulus μ, and density D, etc., are uniform for the material.

The boundary of the solid is an arbitrary convex surface assumed to be given. There are no macroscopic voids within that boundary. Any point in the solid has a position given by the vector *

$$\boldsymbol{r} = \boldsymbol{i}x + \boldsymbol{j}y + \boldsymbol{k}z, \tag{24}$$

when there is no elastic strain. In the absence of strain the solid is at rest with respect to the coördinate axes X, Y, Z fixed in an inertial system. An arbitrary small strain moves any point of the solid a vector distance

$$\boldsymbol{\varrho} = \boldsymbol{i}\xi + \boldsymbol{j}\eta + \boldsymbol{k}\zeta, \tag{25}$$

where ξ, η, ζ are functions of x, y, z and time t. The strain is due only to forces acting at the boundary.

Under these assumptions, the volume density of the elastic force and the resulting acceleration at any point in the solid are given by †

$$\boldsymbol{F} = (k + 4\mu/3)\nabla\nabla\cdot\boldsymbol{\varrho} - \mu\nabla\times\nabla\times\boldsymbol{\varrho} \tag{26}$$

$$= D\frac{\partial^2\boldsymbol{\varrho}}{\partial t^2}. \tag{27}$$

The absolute magnitude of the maximum positive tension p at a point can always be written in the form‡

$$p = (k - 2\mu/3)\nabla\cdot\boldsymbol{\varrho} + 2\mu(\boldsymbol{n}\cdot\nabla)(\boldsymbol{n}\cdot\boldsymbol{\varrho}), \tag{28}$$

where $\boldsymbol{n}$ is the unit normal vector of the surface of positive tension. Since this is arbitrary in orientation, all $\boldsymbol{n}$ is given by points on a unit sphere about any point:

$$\boldsymbol{n}\cdot\boldsymbol{n} = 1. \tag{29}$$

By proper local orientation of the axes, the elastic potential

* The notation used here is identical with that of L. Page, "Intr. to Theoret. Physics," Chap. III; Van Nostrand, N. Y., 1928.

† L. Page, pp. 132–54.

‡ L. Page, pp. 139–40. The equations there are not written in terms of the directional derivative $\boldsymbol{n}\cdot\nabla$.

energy density at a point can always be written in the form *

$$E = \tfrac{1}{2}(k - 2\mu/3)(\nabla \cdot \varrho)^2 + \mu\left[\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \eta}{\partial y}\right)^2 + \left(\frac{\partial \zeta}{\partial z}\right)^2\right]. \quad (30)$$

The explicit consideration of the shear strains $\partial\xi/\partial y$, etc., is thus eliminated, for these are of interest only insofar as they contribute to the rupturing stress p.

If now the positive tension p be non-uniform at any time short of fracture, let its maximum value be $u \pm \delta p$ where $\delta p \boldsymbol{n}$ is a small positive tension. If by chance δp vanishes and then becomes negative, an initiating failure will probably result at that point rather than elsewhere by definition of u. If $p = u \pm \delta p$ at all points, then as δp decreases and becomes negative it is reasonable to suppose that an initiating failure is as likely at one point as at another. The condition for initiating failure to be equally likely at all points and take any possible orientation is thus

$$p = u \pm \delta p, \qquad \iiint \boldsymbol{n} \delta p \, dv = 0. \quad (31)$$

The condition that $\sum \delta p \boldsymbol{n}$ be zero on the average is a recognition of the fact that chance fluctuations of stress may cause failure even when $p \ll u$. The vector $\boldsymbol{n}$ gives the orientation of the initiating element of failure. Letting $\boldsymbol{n}$ take all directions within the possible 4π radians

$$(\boldsymbol{n} \cdot \nabla)(\boldsymbol{n} \cdot \varrho) = \frac{\partial \xi}{\partial x} = \frac{\partial \eta}{\partial y} = \frac{\partial \zeta}{\partial z} = \nabla \cdot \varrho / 3. \quad (32)$$

Substituting in equation (28) and neglecting δp for the average case

$$\nabla \cdot \varrho = u/k, \quad (33)$$

whence

$$\nabla \nabla \cdot \varrho = 0; \quad (34)$$

integrating (33) with respect to $\boldsymbol{r}$ at any time

$$\varrho = \frac{u}{3k} \boldsymbol{r} + \varrho_0(t), \quad (35)$$

whence

$$\nabla \times \varrho = 0, \quad (36)$$

* L. Page, p. 141; with certain modifications.

$$\nabla \times \nabla \times \varrho = 0 \tag{37}$$

and

$$\boldsymbol{F} = \frac{\partial^2 \varrho}{\partial t^2} D = 0. \tag{38}$$

We conclude that the average solid exhibits no wave motion prior to the time of failure. On the average, the strain is irrotational at all points.

The resultant dilatation is accompanied by an average positive normal displacement ρ_n over the surface. All displacements being small, the initial volume V_0 and boundary surface S_0 are sensibly unchanged. We have always

$$\iiint \nabla \cdot \varrho dv = \iint \varrho \cdot n dS_0$$

and here

$$\rho_n = u V_0 / k S_0. \tag{39}$$

Integrating equation (38) with respect to time

$$\varrho = t\dot{\varrho}(x, y, z) + \varrho_0(x, y, z, t). \tag{40}$$

The time rate of elastic displacement $\dot{\varrho}$ of any point of the solid, and therefore of the boundary, is a function of position only. It is clear that any acceleration of displacement would result in a concentration of stress which might locally exceed the ultimate, making failure more likely at such points. Impulsive point forces at the boundary do not satisfy this condition for fracture stresses would then be more likely at the boundary than elsewhere. When $\dot{\rho}$ is very small compared with the velocities of propagation * $\sqrt{(k + 4\mu/3)/D}$, $\sqrt{\mu/D}$, i.e., when the solid is very rigid and resistant to compression, stress concentrations due to small accelerations of ϱ do not tend to occur. In impact and jaw crushing mechanisms the velocities of motion are fairly low compared with those of propagation in most brittle materials. The tendency to fracture at preferred points in the solid by such mechanisms is due rather to the size-shape selective action of the crusher.

Integrating equation (38) throughout the volume at time t

$$\iiint \boldsymbol{F} dv = \iint p n dS_0, \tag{41}$$

where $p(x, y, z, t)$ is the positive tension at the boundary.

* L. Page, pp. 152–58.

The resultant of the boundary forces must therefore vanish at all times for the average solid. By (41)

$$\boldsymbol{F} = \nabla p = 0$$

or, integrating

$$p = p(t). \tag{42}$$

The average positive boundary tension is therefore spatially uniform. For example, we may picture a sphere compressed now along one random diameter, and now along another. The average tension for an indefinitely large number of spheres is positive and can be represented by one with uniform positive tension over its surface.

Assuming the coördinate system to be attached to the solid at three points, the latter has a kinetic energy with respect to that coördinate system which is due only to the small velocities $\dot{\varphi}(x, y, z)$. Strains in the absence of external forces being small and wave motion absent we neglect all initial energy stored in the average solid. The average external forces now act and prior to the moment of fracture do work W_0 in amount. Since they act slowly, very nearly all of W_0 is accounted for in the elastic potential energy at the moment of fracture. Substituting (32) and (33) in (30), the average density of elastic energy at this moment is a constant of the material

$$W_0/V_0 = u^2/2k. \tag{43}$$

The rate at which the boundary pressure does work on the solid at any time is

$$p(t)\dot{\rho}_n S_0,$$

where $\dot{\rho}_n$ is the instantaneous surface average of the normal rate of boundary displacement. By (40) $\dot{\rho}_n$ is not a function of time. If τ be the average interval prior to fracture, the average tension p_a during that interval is given by

$$W_0 = p_a \dot{\rho}_n \tau S_0 = p_a \rho_n S_0,$$

whence by (39) and (43)

$$p_a = u/2. \tag{44}$$

The time-space average of the absolute boundary force at a

physical point is thus

$$u\Delta^2/2 = \text{const.} \tag{45}$$

To summarize the necessary average conditions for a random distribution of initiating fractures: within the boundary, a homogeneous pure tension strain just sufficient to produce tensile failure; at the boundary, a spatially uniform negative pressure, whose time average for the pre-fracture period is half the tensile strength of the material. The characteristic rate of boundary displacement is assumed small compared with the propagation velocities of the material. The mean normal surface displacement in the average fracture period is smaller for weak rigid solids which expose a large amount of surface per unit volume. These conclusions are not novel but, in the past, elastic solid theory has concerned itself not with haphazard but with organized boundary conditions. The emphasis here falls upon a random set of boundary forces acting on an ensemble of solids. The next step is to define such a set of forces and show that, on the average, they are capable of producing the conditions required by randomness of initial fracture.

The most general force at any point of the boundary may be represented by tangential and normal vectors which are functions of position and time. But frictions are here absent, hence the tangential component must vanish. The normal component is then inward and has the arbitrary orientation $-\boldsymbol{n}$, subject only to:

$$\boldsymbol{n}\cdot\boldsymbol{n} = 1$$

and may be written at any time or place in the form

$$-\boldsymbol{n}F_i(x, y, z).$$

If we take the time average of the point force over the period prior to the moment of fracture, we can write for its average at a point

$$-\boldsymbol{n}F(x, y, z).$$

For the whole surface there are γ such physical point forces. Any or all may take absolute values from zero upward prior to the moment of fracture, for the negative tension may take

any value whatever so long as contingent positive tensions do not exceed u.

There is therefore an indefinite number of vectors $-nF$ which represent the possible action on the surface prior to fracture. These vectors are the time averages of the chance forces which supply the strain energy involved in the initiation of fracture. They are here to be subjected only to the condition (41) that they be in equilibrium on the average, and the condition (45) that their time and space absolute average during a fixed period be constant. A Maxwellian distribution of F has this property as will now be shown. It is further supposed only that the chance of a given magnitude F depends upon itself and the volume of the force-space element in which the vector $-nF$ terminates. All magnitudes $0 < F < \infty$ are admissible, and all directions n are equally likely. It follows from several well-known courses of reasoning [6] that the chance of a force F_x, F_y, F_z is

$$\phi_x\phi_y\phi_z = \frac{1}{\beta^3\pi^{\frac{3}{2}}} e^{-(F_x^2+F_y^2+F_z^2)/\beta^2}, \tag{46}$$

where β is a force magnitude independent of position and time. The resultant or mean force in any direction, as, X, is

$$\int_{-\infty}^{\infty} F_x\phi_x dF_x = 0, \tag{47}$$

which satisfies the equilibrium condition (41). The average absolute boundary force is the same for all directions, as X,

$$\int_{-\infty}^{\infty} F_x^2\phi_x dF_x = \beta^2, \tag{48}$$

which satisfies the absolute average condition (45). The condition that the average boundary forces be chaotic is thus sufficient for the production of random initiating failures in the average solid.

To produce random fracture, the random forces γ in number, β in intensity, must satisfy the boundary conditions within permissible limits as measured by h. β being arbitrary, let the typical point force be defined by

$$\beta \equiv \tfrac{1}{2}u\Delta^2. \tag{49}$$

γ must be such that equilibrium is preserved. Thus at very least $\gamma = 2$. If $\gamma = 2$ for the average solid, the direction of the action is that of the surface normal $\boldsymbol{n}$ and passes through the center of gravity. $\boldsymbol{n}$ being arbitrary the representative solid is a sphere. Let the diameter be L. Then

$$\left.\begin{aligned} V_0 &= \pi L^3/6 \\ S_0 &= \pi L^2 \\ V_0/S_0 &= L/6. \end{aligned}\right\} \tag{50}$$

But two random forces are unlikely to retain the equilibrium of the representative sphere. At least 4 forces will become active before deformation can initiate, therefore

$$\gamma \geqq 4. \tag{51}$$

Deformation must proceed at a velocity negligible in comparison with those of propagation. Taking the speed of the transverse disturbance as the standard since this is smaller than the longitudinal, let

$$\dot{\rho}_n = \frac{1}{h}\sqrt{\mu/D}. \tag{52}$$

The work of random fracture being W_0, the actual work must be done at a rate corresponding to $\dot{\rho}_n$, β and γ as so far defined,

$$W_0/\tau = \dot{\rho}_n \beta \gamma. \tag{53}$$

Noting that $\dot{\rho}_n \tau = \rho_n$ and substituting (43), (49) and (50)

$$\gamma = h^2 \pi. \tag{54}$$

Whence the sensibility with which the process is viewed is subject to $h^2\pi \geqq 4$, or, as anticipated in the paragraph on homogeneity, eq. (23),

$$h \geq \sqrt{4/\pi}. \tag{55}$$

The restriction on the magnitude of the point boundary force is, by (49), (50) and (55),

$$\beta = uL^2/2h^2 \leq uS_0/8 \tag{56}$$

and on the velocity

$$\dot{\rho}_n \leq \sqrt{\pi\mu/4D}. \tag{57}$$

By (50), (55) and (56) the average prefracture period becomes

$$\tau = \rho_n/\dot{\rho}_n \geq \frac{2uV_0}{kS_0}\sqrt{D/\pi\mu} = \frac{u}{3k}L\sqrt{D/\pi\mu}. \qquad (58)$$

Since all systems of measurement distinguish more than one category of position within a solid larger than the physical point, the restrictions (56) and (57) on the forces are general. For any system of forces producing random fracture the work done is thus unrestricted, for

$$W' = (\leq uS_0/8)(\geq 4)\rho_n$$

or

$$W' \gtreqless W_0. \qquad (59)$$

A concentrated pressure on the boundary is accompanied by an equal tension parallel to the surface at the point. That a point force will produce fracture is thus more nearly certain as the local pressures approach u, or, as the time space average approaches $u/2$. Hence, on the average, the prefracture pressure is $u/2$. It is practically certain that the arbitrary convex solid will require 4 point forces for fracture. The necessary displacements being small it is unlikely that more than 4 forces will come to bear. It follows that *the average work of random fracture tends to equal* W_0. *It is not necessary that the work done be* W_0. It is sufficient merely that the forces and velocities satisfy the loose restrictions (51), (52) and (56), while performing any amount of prefracture work W'. Indefinitely many classes of mechanically induced fracture conform to these requirements, but the existence of a fracture system in which the fissuring is random does not of course warrant the assumption that it resulted from random boundary forces, much less that the actual work was W_0. The test of randomness lies in the distribution of new surface, the calculation of which now follows.

DISTRIBUTION OF NEW SURFACE.

Let the grain of the typical solid have a statistically uniform surface q. The grain being small in comparison with the physical point, the differential element of boundary surface dS contains many surface quanta dn_s,

$$dS = q\,dn_s. \qquad (60)$$

When an elementary fissure is produced as a result of fracture at a constant temperature far from the melting point, let the corresponding work done in separating the elementary new surfaces be

$$dW = luds/2 \tag{61}$$

per element, where l is a statistically uniform displacement of fracture for that material. Its dependence upon the temperature as that of u does not concern us here except insofar as we avoid material in a critical state.*

The energy dW has for its sources the work dW' done at boundaries and the initial random strain energies within the physical point. Since the new surface of fracture may in general involve amounts of work small in comparison with the work done at the boundaries, the possible contribution dH_0 of the random strains is written into the energy balance of the average solid

$$dW = dW' - dH + dH_0, \tag{62}$$

where dH is the amount of energy entering non-new surface categories at the instant in question.

By reason of the relaxation of minute strains, gas adsorption, etc., the possibility that the production of new fracture surface can proceed reversibly at temperatures well below the melting point is exceedingly small.† That is, on the scale of observation $\Delta \times \Delta \times \Delta$,

$$dS \geq 0. \tag{63}$$

The initiation of fracture is accompanied by a rapid redistribution of the large scale strains and the production of new

* In a heterogeneous material u is a quasi-surface tension which depends upon the actual surface tension of the material and the structure. If W, the actual new-surface energy, is measured statistically, dimensional homogeneity requires a length-dimensioned parameter l in (61). The factor $\frac{1}{2}$ is included because attention is on the boundary surface $2dS$ which is taken to define a single elementary fissure.

† Pursuing the relation to the Second Law further, assume that $luq/2$ is linear with the absolute temperature T. Then $luq/2 = bT$. By (60, 61, 63) $bdS = qdW/T$ so that the increase of new surface is proportional to a quantity whose interpretation is entropy. It also follows that the tendency for fracture surface to appear is greater at lower temperatures. The form of the gas law is noted in $luq = 2bT$.

degrees of freedom. The relaxation of strain and the gross motion of the fragments generally carries the boundaries out of reach of the slowly acting crushing mechanism. The dispersion of energy into new categories reduces the average tension. Since the maximum was $u \pm \delta p$ at all fracture points the chance of further fracture decreases rapidly. The new surface therefore tends to reach a stationary state in an interval very short compared with the prefracture period. That is, the average equilibrium state occurs at

$$\left.\begin{aligned} t &= \tau, \\ dW' = dH_0 = dW = dH &= 0, \\ dS = q\,dn_s &= 0, \\ dn_s &= 0. \end{aligned}\right\} \tag{64}$$

Integrating from the time of application of the boundary forces to time τ for the average solid

$$S = qn_s, \tag{65}$$

$$W = \tfrac{1}{2}luS, \tag{66}$$

$$W' = W + H - H_0. \tag{67}$$

The absolute amount of new surface produced is then

$$S = \frac{2}{lu}(W' + H_0 - H). \tag{68}$$

The action at the boundaries thus not only supplies work of new surface, but may also liberate by trigger action an indefinite quantity of random strain energy. This, while small, may be comparable with the work of new surface and partially accounts for the ordinarily low correlation [7] between S and W'. It is clear that this correlation should be positive and large for solids weak in tension, and be greater if initial random strains are present. While H is theoretically measurable the difficulty of estimating H_0 will make it necessary to rest content with rough empirical relations in many instances.

In order to determine the manner in which the work and the parameters of the material enter the distribution functions, we make the simplifying assumption, seldom warranted for actual solids as more than an approximation

$$H_0 = 0 \tag{69}$$

and define the efficiency [7]

$$\epsilon \equiv \frac{W}{W'} = \frac{W' - H}{W'}. \qquad \epsilon \neq 0 \quad (70)$$

Reassembling the fragments of the average solid, the sensible conservation of mass, volume and form gives a fractured solid of sensibly the same volume and form as the original. If the work W' is the result of a system of forces satisfying the randomness conditions, the initiating fractures have random-distribution and orientation. As will be shown later, the same is true of the new surface generated. This is consistent with the conclusion that n_s is a constant for the ensemble.

The resulting distribution of fragment diameters is calculated as follows. Consider a circular cylinder, diameter Δ, length s. Let one end coincide with the boundary of a random fragment and the axis have random orientation n_1. The probability that the cylinder contains new surface is independent of its position and orientation provided it lies wholly within V_0, being simply the geometrical probability $\pi\Delta^2 s/4V_0$. The expected number of quanta in the cylinder is then $n_s\pi\Delta^2 s/4V_0$. By (65), (66) and (70) this becomes

$$\frac{\pi\epsilon W'}{2luqV_0}s. \qquad s \gg f\Delta$$

In characterizing the grain we can let its mean shape and diameter be such that

$$q = (f\Delta)^2. \quad (71)$$

Therefore

$$\frac{\pi}{2}\frac{\epsilon W'}{V_0}\frac{s}{luf^2} = \frac{s}{\bar{s}}.$$

Whence

$$\bar{s} = \frac{2V_0}{\pi\epsilon W'}f^2ul, \qquad \epsilon \neq 0 \quad (72)$$

in which the interpretation of $\bar{s}$ has been anticipated in adopting the notation of the distribution functions. The probability of finding exactly n quanta in the cylinder is given by the Binomial Law.[1] When the possible number of events n_s is large and their probability small as here, the

Binomial reduces to the Poisson Law, equation (1). The chance of finding no quanta in the cylinder is the chance that one or more are encountered when its length is greater than s. But the existence of any number of quanta, say $\pi\Delta^2/4q$ beyond s, implies that the random fragment has a random diameter s or greater. Therefore, the oversize distribution of random fragment diameters is

$$\nu(>s) = N_0 e^{-s/\bar{s}}, \qquad s \gg f\Delta \quad (73)$$

where N_0 is the number of fragments and $\bar{s}$ their random mean diameter. The mean volume of the fragment is that of a sphere of the same diameter:

$$\bar{v} = \pi\bar{s}^3/6 = V_0/N_0. \quad (74)$$

The mean fragment surface is then

$$\bar{S} = (S + S_0)/N_0 \quad (75)$$

and the surface per unit volume of the mean fragment $\bar{S}/\bar{v}$. Therefore by (63)

$$d(\bar{S}/\bar{v}) = dS/V_0 \geq 0. \quad V_0 > f^3\Delta^3 \neq 0 \quad (76)$$

Consequently the mean fragment tends toward a configuration having maximum surface per unit volume. But since the fragment tends to be equidimensional, the average fragment is an equidimensional irregular polyhedron whose form approaches that of the cube.

It follows from (73) that the expected number of fragments $\delta\nu(s)$ with diameters in the category $(s, s + f\Delta)$ is

$$\delta\nu(s) = N_0 e^{-s/\bar{s}} f\Delta/\bar{s}. \qquad s \gg f\Delta \quad (77)$$

This agrees with Martin's empirical findings on the random diameters of sand particles.[8, 9] The found relation also held for $s \to f\Delta$ upon successive increases in the power of the microscope under which the fragments were measured. Martin was inclined to extrapolate (77) to include indefinitely finer particles. He was conscious of the choice of discrimination permitted the observer in viewing his material and spoke of " psychological errors " of observation. Our present viewpoint acknowledges this choice explicitly and permits extrapolation through the inclusion of the indefinite fineness factor

$f\Delta$, but also urges the necessary caution in taking that step. If, however, the law holds at one fineness, it holds at all coarser levels.

It is readily verified that the relative weight of material $\delta w(s)/w_0$ of diameter $(s, s + f\Delta)$ is given by the unimodal distributions*

$$\delta w(s) = w_0 \left(\frac{s}{\bar{s}}\right)^a e^{-s/\bar{s}} f\Delta/\bar{s}. \quad a = 1, 2, 3 \quad (78)$$

If (δ, s) be plotted on double logarithmic coördinates, this function is linear for small sizes, and convex upward for large sizes. The slope a is 1, 2 or 3 for fragments resulting from the rod, sheet or three-dimensional materials respectively. For small sizes

$$\delta w = As^a. \quad (79)$$

The expressions (78), (79) agree in form with empirical distributions obtained by Gaudin, with the difference that a is non-integral.[10]

The size-distribution of a mixture of three-dimensional components from different fracture sources is described by a series of exponential terms. The formulæ developed in the first section apply here with the difference that statistically homogeneous segments of a thin cylinder take the place of the linear segments of that analysis. The cylindrical subregion of our analysis gives at once the case of the thin rod of length L if all material outside its boundaries be removed.

It may be noted that the distribution functions apply whenever the new surface is random, but the parameter $\bar{s}$ does not then necessarily have the mechanical interpretation (72). Thus Martin's and Gaudin's materials resulted from a complex of conditions only partly analyzed here, though their findings are anticipated as to general form.†

* The mass distributions of colloid particles calculated from thermodynamic considerations have an analogous form. See N. and E. v. Rashevsky, *Z. f. Phys.*, **46Bd.**, 3, 4, p. 300, 1927.

† Preliminary study of bubble sizes in a foam suggests similar relationships.

(*To be continued.*)

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

UNITS OF WEIGHT AND MEASURE.

Definitions and tables of equivalents of the customary weights and measures of the United States and the metric system are included in Miscellaneous Publication M121, released last month.

This compilation supersedes Bureau of Standards Circular C47, issued in 1914, which has been out of print for some time. Necessary corrections have been made and the text revised and rearranged. The new publication contains the table of millimeter-inch equivalents formerly printed as a supplement to C47; the chart showing a graphic comparison of screw-thread pitches, originally published as Miscellaneous Publication M49; and the metric-English equivalents for athletic events which have heretofore appeared as Letter Circular LC376.

The fundamental equivalents and the definitions of the units are clearly set forth, and the difference between units and standards is briefly explained. The methods adopted for maintaining the standards with the required degree of precision are described.

The tables are arranged for maximum convenience in use, and the paper and type were selected with great care to insure easy reading. An index adds greatly to the utility of the publication.

Copies of M121 are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 15 cents each.

A HIGH VERTICAL BROADCAST ANTENNA.

RP874 in the April number of the *Journal of Research* describes experiments made during December, 1934, to determine the relative performances of an inverted L antenna and a high

* Communicated by the Director.

vertical mast antenna in broadcasting from station WBT, Charlotte, N. C. Continuous field intensity records of emissions from each of these antennas were made at seven different distances varying from 69 to 879 kilometers. Also field intensity measurements were made at eight points on a circle one mile from the transmitting station. The latter measurements indicated that the field intensity at one mile was almost doubled by the substitution of the high mast for the inverted L antenna. The field intensity records indicated that the same antenna change increased the field intensity at most of the distant points about 1.5 times, and reduced the amplitude of the fading considerably at the first three receiving points, 69 to 142 kilometers distant.

With either antenna the frequency of the fading was considerably greater at the first three receiving stations than at the more distant ones. This rapid type of fading seemed to be produced largely by interference of ground and sky waves, both being appreciable components of the received fields at the first three receiving stations. The frequency of the fading was not appreciably changed by the substitution of the mast for the L antenna.

During part of this experiment four wires with a spacing of 90 degrees were dropped from cross arms at the top of the mast to increase its top capacity. This arrangement did not appreciably increase the field intensities at one mile or greater distances but decreased the fading at the first three receiving stations.

EVALUATION OF ULTRAVIOLET SOLAR RADIATION OF SHORT WAVE-LENGTHS.

In RP877 in the April number of the *Journal of Research*, technical details are given for evaluating the ultraviolet of short wave-lengths in solar radiation by two closely agreeing methods: (1) by means of a balanced thermopile and filter radiometer which is calibrated against a standard of thermal radiation; and (2) by means of a titanium photoelectric ultraviolet intensity meter calibrated in absolute value against a standard of ultraviolet radiation.

Both methods of evaluation require a knowledge of the distribution of energy in the ultraviolet spectrum. A

method is described for determining the spectral energy distribution in the extreme ultraviolet by means of a titanium photoelectric cell and four filters, thereby eliminating one of the uncertainties in the evaluation of ultraviolet radiation in absolute units.

The intensity of ultraviolet radiation, of wave-lengths shorter than and including 3132A, in the clearest midsummer midday sunlight, in Washington is about 75 microwatts per cm.^{2}, decreasing to one-tenth this value (8 microwatts per cm.^{2}) during the clearest midwinter midday sunshine.

Data are given on ultraviolet solar intensities observed in Washington, D. C., San Juan, P. R., and Flagstaff, Arizona.

Extrapolation of the data observed at the three stations (Washington, San Juan, and Flagstaff) indicates an ultraviolet intensity of about 600 microwatts per cm.^{2}, outside the earth's atmosphere—a 5 to 8 fold increase as compared with a 20 to 30 per cent. increase in total intensity of all wave-lengths.

SIMPLIFIED APPARATUS FOR TECHNICAL SUGAR COLORIMETRY.

The sugar technologist in order to meet the demands of commerce and to control properly his processes of manufacture must be able to measure sugar color accurately and to express the results of measurement in terms that have a rational meaning and that may be of practical value when included in the computations involved in the scientific control of sugar manufacture. Until about 1920 the only method of sugar colorimetry was that devised by Carl Stammer, an eminent German sugar technologist, and described by him in 1860. In the Stammer colorimeter the attempt is made by measurably varying the depth of a sugar solution to match the intensities of the light transmitted by the solution and by a standard composed of a yellow glass plate, daylight or other white light being the source of illumination. Unfortunately the spectral transmission and quality of color of Stammer glasses or of any other yellow glass are never the same as that of sugar solutions, and accurate matching is impossible. Various substitutes for the yellow glasses have been proposed but none of them, for one reason or another, has been found reliable. As a result of investigations at the Bureau, which

are described in the April number of the *Journal of Research* (RP878), a method has been developed for accurately matching these intensities. Instead of employing an incandescent light source giving the entire visible spectrum, an isolated and relatively narrow portion of the spectrum is used in such a way that light of virtually one effective wave-length reaches the eye after passage through sugar solution and transmission standard. By choice of the proper wave-length the color of the solution may be measured. A simple way of accomplishing this is described in the paper. The comparator is an instrument of standard design illuminated with an incandescent tungsten source from the light of which narrow spectral bands are isolated by means of special filters. Comparison is made with a glass plate calibrated in terms of transmission.

EFFECT OF HYDROGEN PEROXIDE ON WOOL.

In practical wool processing such as bleaching, carbonizing, lustering, printing, chlorination and bromination for producing "unshrinkable" finishes, and in natural aging involving the action of light, air, and moisture, wool may become partially oxidized, the extent of the oxidation depending on the severity of the treatments. The quantitative estimation of the oxidation is of great importance, since previous work has shown that this oxidation increases the susceptibility of wool to deterioration by alkaline treatments.

In the April number of the *Journal of Research* (RP875), an investigation is described, the purpose of which was to provide quantitative data showing the effects of oxidizing agents on wool as a basis for studying all processes in which wool is exposed to oxidation reactions.

Data are presented to show the effect on wool of varying the concentration, temperature, and pH of the hydrogen peroxide solutions, and the duration of treatment. The main point of attack during oxidation is the disulfide group of the cystine in the wool. The extent to which the wool is oxidized bears a functional relationship to the decrease in the cystine content and to the increase in the alkali-solubility of the wool.

LEAD ACETATE TEST FOR HYDROGEN PEROXIDE TREATED WOOL.

There has been a definite need for a qualitative test for distinguishing between untreated wool and wool bleached with hydrogen peroxide. Tests on the physical properties have been shown to be useless, while dyeing tests are only partially successful, since they were designed to measure damage to wool in general, and lack specificity. The lead acetate test has been used by various workers but the method as outlined is unsatisfactory since it fails in the presence of small amounts of acid and alkali.

In an investigation described in the April number of the *Journal of Research* (RP876), the degree of darkening of wool by the formation of lead sulfide during treatment with lead acetate solutions of varying pH was studied. It was found that a maximum difference between untreated and bleached wool occurred at pH 5. The results indicate that hydrogen peroxide partially oxidizes the sulfur in wool and prevents the formation of lead sulfide. A mechanism by which lead sulfide is formed in untreated wool is described.

CONSISTENCY OF ENAMELS AT FIRING TEMPERATURES.

In Technical News Bulletin 223 (November, 1935) a description was given of the apparatus used at the Bureau for determining the consistency of molten enamels. It was also stated that opaque covercoat frits, which contain suspended matter in the glassy matrix, act as non-Newtonian fluids (neither viscous nor plastic) at and near the firing temperatures. This fact complicates the numerical reporting of results, both because the apparent viscosity of this type of material varies with the rate of shear, and also because it is to some extent dependent upon the apparatus with which it is measured. The following values of apparent viscosity (which may be subject to some revision) are for the previously described apparatus, the rates of shear being such as to produce a constant torque on the suspension wire, causing a 5 degree twist. These qualifications are not necessary for the numerical expression of the results on ground coats, since they act as viscous fluids.

Type of Sheet-Iron Enamel.	Apparent Viscosity in Poises, Temperature, °C.				
	750	800	850	900	950
"Soft" ground coat	26,800	6,200	1,900	600	200
"Hard" ground coat	96,600	23,800	6,600	2,300	900
"Regular" cover coat	13,200	5,100	2,200	1,100	540
"Very opaque" cover coat	69,200	19,700	6,500	2,500	1,160
"Acid resisting" cover coat	———	109,600	26,000	7,700	2,900

It will be noted that there is a large difference in the consistencies of the different enamels. Under the conditions specified above, the acid-resisting cover coat enamel requires at least 100° C. higher temperature than the "ordinary" cover coat, to have the same order of apparent viscosity. The "hard" ground coat requires about 50° C. higher temperature than the "soft" one. Also, it will be noted that the relative order of the apparent viscosities of the enamels is different at different temperatures.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, APRIL 15, 1936.

The regular stated meeting of the Institute was called to order by Mr. Henry Butler Allen, Secretary and Director, at 8:30 o'clock. He announced that the following members had been added to the roll since the last report: Institute members—31; Museum members—62; total—93.

He also announced the death of Mr. Richard L. Binder, the donor of the Foucault Pendulum which he installed in the museum, and that of Mr. H. W. Breyer, a member for many years.

Announcement was made of the final meeting of the Institute on which occasion the medals of the year would be awarded. This meeting will be held on Wednesday afternoon, May 20, at 3:30 o'clock. The recipients of the Franklin medal are Dr. Frank Baldwin Jewett and Mr. Charles Franklin Kettering.

Mr. Allen also called attention to the special showing of the Chalice of Antioch in Franklin Hall, which will be continued until May 3.

Mr. Clarence A. Hall, Chairman of the Bartol Research Foundation Committee was then requested to take the chair. Mr. Hall introduced Dr. W. F. G. Swann, Director of the Bartol Research Foundation who presented an account of the activities of the Foundation during the past year. He spoke especially of the progress made in researches upon nuclear physics and cosmic rays. He also gave an account of the cosmic ray results obtained on instruments and apparatus made in the laboratory of the Bartol Research Foundation and used during the recent National Geographic U. S. Army Air Corps stratosphere flight. Reference was also made to airplane flights of the future and balloon equipment for altitude measurements.

A discussion followed the reading of the report, after which a rising vote of thanks was extended to the speaker. Adjourned.

HENRY BUTLER ALLEN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, April 8, 1936.)

HALL OF THE COMMITTEE,
PHILADELPHIA, APRIL 8, 1936.

MR. JOHN A. VOGLESON *in the Chair*

The following reports were presented for final action:

No. 2993: Sprayed Metal Coating Process.

This report called attention to the death of Mr. R. L. Binder on March 20, 1936, which automatically removes the case from further consideration.

No. 3003: Work of Dr. G. O. Curme, Jr.

This report recommended the award of the Elliott Cresson Medal to Dr. George O. Curme, Jr., of New York City, "In consideration of his foundation of

a new technology which has resulted in the development of a rapidly expanding industry, namely, that of synthetic aliphatic compounds based upon the olefines as starting material; and his development of profitable uses for these compounds in industry."

The following report was presented for first reading:

No. 3008: Oilostatic Transmission System.

GEO. A. HOADLEY,
Secretary to Committee.

LIBRARY NOTES.

RECENT ADDITIONS.

American Foundrymen's Association. Transactions 1935. Volume 43. 1936.

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COLLINS, A. FREDERICK. Fun with Electricity. A How-To-Make-It Book of Simple and Startling Experiments with Direct, Alternating, and High Frequency Electric Currents. 1936.

DOHERTY, ROBERT E., AND ERNEST G. KELLER. Mathematics of Modern Engineering. Volume I. 1936.

EBERHARD, G., A. KOHLSCHÜTTER, AND H. LUDENDORFF. Handbuch der Astrophysik. Band VII: Ergänzungsband berücksichtigend die Literatur bis Ende 1934 nebst einem Generalregister des Gesamtwerkes. 1936.

ELDRIDGE, E. F., AND F. R. THEROUX. A Laboratory Manual for the Chemical Analysis of Water and Sewage. First Edition. 1936.

Electrochemical Society. Transactions 1935. Volume 68. 1936.

FATH, EDWARD ARTHUR. Through the Telescope: a Story of the Stars. 1936.

FLETCHER, GUSTAV L., IRVING MOSBACHER, AND SIDNEY LEHMAN. Unified Physics: Matter in Motion. 1936.

Forschungsinstitut der čechoslovakischen Zuckerindustrie in Prag. Bericht für das Jahr 1934–1935. XXXVIII. 1935.

FRIEND, J. NEWTON, Ed. A Text-Book of Inorganic Chemistry. Volume XI: Organometallic Compounds. Part III: Derivatives of Phosphorus, Antimony and Bismuth, by Archibald Edwin Goddard. 1936.

HAMMETT, LOUIS P. Solutions of Electrolytes with Particular Application to Qualitative Analysis. Second Edition. 1936.

HEIL, LOUIS M. The Physical World. 1936.

HEILBRON, I. M., AND H. M. BUNBURY. Dictionary of Organic Compounds. Volume Two: Eccaine-Myrtillin Chloride. 1936.

HICKINBOTTOM, WILFRED JOHN. Reactions of Organic Compounds. 1936.

JONES, T. J. Thermionic Emission. 1936.

LINDSAY, ROBERT BRUCE, AND HENRY MARGENAU. Foundations of Physics. 1936.

McLACHLAN, N. W. The New Acoustics. A Survey of Modern Development in Acoustical Engineering. 1936.

MOYER, JAMES A., AND JOHN F. WOSTREL. Radio Receiving and Television Tubes, Including Applications for Distant Control of Industrial Processes and Precision Measurements. Third Edition. 1936.

MYERS, DAVID MOFFAT. Reducing Industrial Power Costs. First Edition. 1935.

NOBLE, ROYCE J. Latex in Industry. 1936.

OSANN, BERNHARD. Moderne Stahlgiesserei für Unterricht und Praxis. 1936.

PUCHSTEIN, A. F., AND T. C. LLOYD. Alternating-Current Machines. 1936.

Royal Agricultural Society of England. Journal. Volumes 95–96. 1934–1935.

Royal Society of London. Philosophical Transactions. Series B: Biological Sciences. Volume 225. 1935.

SHULL, A. FRANKLIN. Evolution. 1936.

Society of Chemical Industry. Reports of the Progress of Applied Chemistry. Volume 20, 1935.

STALEY, W. W. Mine Plant Design. First Edition. 1936.

STOKLEY, JAMES. Stars and Telescopes. 1936.

SUTHERLAND, G. B. B. M. Infra-Red and Raman Spectra. 1935.

THORPE, JOCELYN FIELD, AND M. A. WHITELEY. Thorpe's Dictionary of Applied Chemistry. Supplement. Volume III: Glossary and Index. 1936.

U. S. Patent Office. Index of Patents Issued from the United States Patent Office 1935. 1936.

U. S. Patent Office. Index of Trade-Marks Issued from the United States Patent Office 1935. 1936.

VENING MEINESZ, F. A. Gravity Expeditions at Sea 1923–1932. Volume 2: Report of the Gravity Expedition in the Atlantic of 1932 and the Interpretation of the Results. 1934.

WAKSMAN, SELMAN A. Humus: Origin, Chemical Composition and Importance in Nature. 1936.

ZIMMER, ERNST. The Revolution in Physics, with an Introduction by Max Planck; Translated, and with a Preface, by H. Stafford Hatfield. 1936.

BOOK REVIEWS.

OLIVER EVANS, A CHRONICLE OF EARLY AMERICAN ENGINEERING, by Greville Bathe and Dorothy Bathe, 362 pages, portraits, plates, illustrations, 24 × 28.5 cms. Philadelphia, The Historical Society of Pennsylvania, 1935. Price $12.00.

Before the birth of this nation Oliver Evans, an engineer, whose work is known to a comparatively small number, was building machinery, serving his country and laying one of the foundation stones for our industrial life.

In "Oliver Evans," which Greville and Dorothy Bathe call "A Chronicle of Early American Engineering," the writers have presented a chronology of incidental events from Evans' birth in 1755 to his death in 1819, the same year as that of James Watt.

This is a very refreshing depiction of the man's work, his efforts and fights and litigations in marketing inventions that were necessary to the entire country,

from Washington down. It is also a departure in biographies from the time-worn chapterization. The Bathes have carried us along with Evans year by year; told us details of his many inventions; shown us drawings, photographs and letters, revealed intimate phases in his business life; and finally have proven to us, when there is no bibliography for reference, the best thing to do is to set one up. They have built a readable, interesting story based on the investigations of years, from Michigan to Great Britain, of the great majority of known manuscripts, drawings, landmarks, governmental records; and in doing this work, have presented to the engineering profession knowledge and facts concerning a man who outranks many whose names are sung more loudly, by far, than Evans.

Evans lived during a time when inter-changeability had its birth in Simeon North's gun factory at Middletown, Conn., when machine tools were crude or nearly non-existent, when a 200 lb. boiler was far more risky than Moultrops' 1600 lb. experiment of a century after Evans had gone, and when film lubrication had not reached the point to which Kingsbury brought it. But Evans pioneered in steam, manufacturing and bearings just as our present-day students are doing —all under the adverse conditions of the eighteenth and early nineteenth centuries—and the Bathes have presented him to us in his proper light. For this Engineering owes them a rising and evergrowing vote of thanks.

As a book, its historic and technical values aside, the Pennsylvania Historical Society has produced a beautiful volume, worthy in every way of a place in a book lovers' collection.

H. S. HARRIS.

BEHOLD! THE GRAND PROBLEM, THE CIRCLE SQUARED BEYOND REFUTATION, NO LONGER UNSOLVED, MATHEMATICAL AND GEOMETRICAL DEMONSTRATIONS, by Carl Theodore Heisel. 330 pages, diagrams, 17.5 × 26.5 cms. Cleveland, Author, 1934.

As the title implies, this book is an exposition of the theory of the squared circle, a theory much in question by mathematicians of various degrees. The author is a successful business man who early in life became interested in the work of Mr. Theodore Faber in this respect. In July 1884 he was instrumental in having Mr. Farber's discoveries published in the International Standard, a monthly magazine of the International Institute for Preserving and Perfecting Anglo-Saxon Weights and Measures. He believes that Mr. Faber has solved the problem of the squared circle and "that his ratios are absolutely true, accurate and incontrovertible, and to be the greatest mathematical discovery in the history of the world. For that reason he is publishing several thousand of these books, at considerable expense to himself, and distributing them free to libraries, colleges and scientists throughout the United States and foreign countries to promulgate the new truth and leave the world better off because he has lived."

"Mr. Heisel has enlarged on Mr. Theodore Faber's discoveries and has carried them on further than Mr. Theodore Faber did himself. He has added a number of diagrams, demonstrations and numerical proofs, and there is no limit to which this great work can be extended and developed in the future by others, in the hope that they will eventually be acknowledged, and the entire mathematical science revolutionized. While he has made many calculations and demonstrations with spheres and cubes to which these ratios are just as harmonious as they are to

squares and circles, he feels that they are foreign to the sole object and purpose of this book, which is to demonstrate and prove the squaring of the circle, and he is leaving the development of other fields and branches of mathematics and geometry to the thoughtful student who prefers to do his own thinking. . . ."

R. H. OPPERMANN.

GRAPHICAL SOLUTIONS, by Charles O. Mackey, M.E. 130 pages, diagrams, 15 × 23 cms. New York, John Wiley & Sons, Inc., London: Chapman & Hall, Ltd., 1936. Price $2.50.

Every practicing engineer, at some time or other, finds the knowledge of graphics very useful as a time saver and helpful in eliminating errors, as well as being of assistance in visualizing equations. There are several well known texts on this subject and their use by graduate engineers is quite frequent. Now the book at hand, according to the author, is not a treatise on graphical computations, instead, it contains the material believed to be sufficient for an elementary course. There are five chapters as follows: Stationary Adjacent Scales; Sliding Scales; Network or Intersection Charts; Alignment Charts; Empirical Equations—Non-Periodic Curves. The author believes that the space allotted to each subject is nearly in proportion to its importance to engineering students and engineers.

The author has made an unusually clear presentation and, as most of the carefully selected exercises illustrating the work can be solved with a working knowledge of logarithms, algebra and plane geometry, the book should have wide usage among technical men.

R. H. OPPERMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS:

Report No. 540, Interference of Wing and Fuselage from Tests of 209 Combinations in the N. A. C. A. Variable-Density Tunnel, by Eastman N. Jacobs and Kenneth E. Ward, 37 pages, plates, tables, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

Tests of 209 simple wing-fuselage combinations were made in the N. A. C. A. variable-density wind tunnel to provide information regarding the effects of aerodynamic interference between wings and fuselages at a large value of the Reynolds Number. This investigation is part of a basic investigation of aerodynamic interference now in progress at the Committee's laboratory and considers the interference as affected by the more important variables of a combined wing and fuselage.

Most of the tests were made with a round fuselage in combination with a rectangular wing of symmetrical section. Variations of the vertical position, longitudinal position, and angular position were covered. A sufficient number of tests of other variables, such as the wing and fuselage shape, were made to give a general understanding of the effects of these variables. For some of the combinations in which the wing and fuselage were not connected, the air forces on the wing and fuselage were determined separately in order to investigate the mutual interference.

The principal results are given in tabular form and summarized by presenting the important characteristics for all the combinations by means of parameters in a single table so that the relative merits of the various combinations may be readily

compared. The results are discussed in relation to the character, cause, and significance of the interference effects encountered under various conditions.

Report No. 541, Aerodynamic Characteristics of Wings with Cambered External-Airfoil Flaps, Including Lateral Control with a Full-Span Flap, by Robert C. Platt, 17 pages, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

The results of a wind-tunnel investigation of the N. A. C. A. 23012, the N. A. C. A. 23021, and the Clark Y airfoils, each equipped with a cambered external-airfoil flap, are presented in this report. The purpose of the research was to determine the relative merit of the various airfoils in combination with the cambered flap and to investigate the use of the flap as a combined lateral-control and high-lift device.

Each of the three airfoils was tested in combination with a flap having a chord 20 per cent. of the main wing chord. The airfoil giving the best characteristics was then tested in combination with a 30-per cent.-c. flap. A satisfactory flap hinge-axis location was selected from the data already obtained and final force and lateral-control tests were made with the 20-per cent.-c. flap hinged at this point. In the lateral-control tests, the flap was cut at the center line of the model so that the semispan flaps could be deflected as ailerons with respect to each other. The flap was also cut at points one-half the semispan from each tip, permitting use of 25 per cent. of the span on each tip as a combined aileron and flap, the center 50 per cent. of the span being used solely as a flap.

Report No. 542, Potential Flow About Arbitrary BiPlane Wing Sections, by I. E. Garrick, 29 pages, tables, 23 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

A rigorous treatment is given of the problem of determining the two-dimensional potential flow around arbitrary biplane cellules. The analysis involves the use of elliptic functions and is sufficiently general to include the effects of such elements as the section shapes, the chord ratio, gap, stagger, and decalage, which elements may be specified arbitrarily. The flow problem is resolved by making use of the methods of conformal representation. Thus the solution of the problem of transforming conformally two arbitrary contours into two circles is expressed by a pair of simultaneous integral equations, for which a method of numerical solution is outlined. It is pointed out that an inverse method of transforming conformally two circles into the wing profiles of a biplane arrangement leads readily to the development of related families of biplane combinations. Flow formulas are developed giving the velocity and pressure at any point of the surface of either profile of the arbitrary biplane arrangement, for any angle of attack. The theory of the monoplane wing section in potential flow is shown to be a degenerate case in which the elliptic functions reduce to trigonometric functions. The general method presented may be employed to determine the potential flow in any doubly connected region and hence may be applied to the single slotted wing or to the auxiliary-airfoil wing.

As an example of the numerical process, the pressure distribution over certain arrangements of the N. A. C. A. 4412 airfoil in biplane combinations is presented and compared with the monoplane pressure distribution.

Report No. 547, Wind-Tunnel Interference with Particular Reference to Off-Center Positions of the Wing and to the Downwash at the Tail, by Abe Silverstein and James A. White, 13 pages, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

The theory of wind-tunnel boundary influence on the downwash from a wing has been extended to provide more complete corrections for application to airplane test data. The first section of the report gives the corrections at the lifting line for wing positions above or below the tunnel center line; the second section shows the manner in which the induced boundary influence changes with distance aft of the lifting line.

Values of the boundary corrections are given for off-center positions of the wing in circular, square, 2 : 1 rectangular, and 2 : 1 elliptical tunnels. Aft of the wing the corrections are presented for only the square and the 2 : 1 rectangular tunnels, but it is believed that these may be applied to jets of circular and 2 : 1 elliptical cross sections. In all cases results are included for both open and closed tunnels.

Report No. 549, Wind-Tunnel Investigation of the Aerodynamic Balancing of Upper-Surface Ailerons and Split Flaps, by Carl J. Wenzinger, 18 pages, tables, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1935. Price ten cents.

An investigation was made in the N. A. C. A. 7- by 10-foot wind tunnel to determine the effectiveness of various methods of reducing the high control forces of unbalanced upper-surface ailerons and of unbalanced split flaps. An unusual test installation was used in which a large-chord wing model was mounted on the regular balance between two large end planes. The tests included many different aileron-axis and flap-axis locations and several arrangements of aileron and flap slots, all on a Clark Y wing. In addition to the hinge moments, the rolling and the yawing moments as well as the lift and drag characteristics of the various arrangements were determined.

As a result of the investigation, a balanced split flap was developed that required control forces about half those of the unbalanced split flap when the balanced split flap was deflected to give approximately the same maximum lift. No entirely satisfactory arrangement of balanced upper-surface ailerons was found but one arrangement was developed that appeared satisfactory except for the stick-force characteristics with small aileron deflections at high angles of attack with the flaps neutral. Another arrangement also appeared satisfactory for all flight conditions except the lowest speeds obtainable with flaps neutral, in which case the indicated rolling moments were low.

THE PHYSICAL WORLD, by Louis M. Heil, Ph.D. 566 pages, illustrations, 14.5 × 22 cms. Philadelphia, P. Blakiston's Son & Co., Inc. 1936. Price $2.75.

Do you wish to learn something about physics, not for the purpose of using it as a basis for going into science, but for cultural purposes only? And do you want to do this in as easy a way as possible? If you do here is the book for you. It is not a text in the strict sense of the word, but rather a simple descriptive treatment that combines astronomy, physics and chemistry mainly. The author begins by explaining the origin and meaning of science, then discusses the external

world—the solar system and the universe. Following this the order of subjects treated is in general similar to what is usually used: Concepts of Matter and Forces; Motion; Structure of Matter; Heat; Sound; Electricity and Magnetism; Light; Transformation of Matter; X-rays and Radio Activity.

Necessarily a book of this nature, in order to accomplish its purpose of facilitating the understanding of the subject, must have an unusual style of writing. This book has. And not only this, but it is evident the author possesses a keen insight into the mind of the non-technical reader so that his work takes advantage of the methods of description which hold interest and stimulate thinking. The result is a remarkably good translation of the technical into the non-technical. In some parts the not over-serious layman reader is even challenged, so to speak, to test his retentive qualities by answering the questions at the end of each chapter. The more serious reader, of course, will not find these very difficult, and will undoubtedly resort to many references listed here to gain more knowledge of certain subjects.

Physics, being an exact science, ordinarily must include mathematics. The lack of mathematics in this book is an outstanding feature. The book is profusely illustrated, each photograph, curve, and diagram being selected with great care to put forth the maximum in coördination between it and the descriptive matter. The subject index completes the book.

There is a great need for everyone to know something more than what is required in his own particular sphere of activity. As time goes on, and life becomes more complicated this is rapidly becoming essential. For the subjects covered, this book is the answer.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Chemical Discovery and Invention in the Twentieth Century, by Sir William A. Tilden, revised by S. Glasstone. 492 pages, plates, 14 × 22 cms. New York, E. P. Dutton & Company, 1936. Price $4.00.

Les Solutions Concentrées, Théorie et Applications aux Mélanges Binaires de Composés Organiques, par Jean Timmermans. 646 pages, illustrations, tables, 16 × 25 cms. Paris, Masson & Cie, 1936.

An Introduction to the Metallurgy of Iron and Steel, by H. M. Boylston, second edition. 563 pages, illustrations, tables, 15 × 23.5 cms. New York, John Wiley & Sons, Inc., London: Chapman & Hall, Ltd., 1936. Price $5.00.

The Chemistry of Natural Products Related to Phenanthrene, by L. F. Fieser, 358 pages, tables, illustrations, 15.5 × 23.5 cms. New York, Reinhold Publishing Corporation, 1936. Price $6.50.

Tables Crépusculaires Donnant l'Altitude au Zénith des Rayons Resants du Soleil Pour Toutes les Latitudes de Degré en Degré, par Jean Lugeon. 438 pages, 20.5 × 26 cms. Warsaw, Drukarnia Panstwowego Instytutu Meteorologicznego, 1934.

Atomtheorie, von Arthur Haas, 284 pages, tables, plates, 16.5 × 24.5 cms. Berlin und Leipzig, Walter de Gruyter & Co., 1936. Price 10 RM.

Transactions of the Bose Research Institute, Calcutta, vol. IX, 1933–34, Biological and Physical Researches, edited by Sir Jagadis Chunder Bose. 210 pages,

illustrations 15 × 23 cms. London, New York, Toronto, Longmans, Green & Co., 1935. Price 18s.

Canada, Dominion Bureau of Statistics, Statistics of Dairy Factories, 1934. 104 pages, tables, 16.5 × 25 cms. Ottawa, King's Printer, 1936. Price twenty-five cents.

The Structure of Metals and Alloys, by William Hume-Rothery. 120 pages, plates, tables, 14 × 21.5 cms. London, The Institute of Metals, 1936. Price 3s.6d.

Canada Department of Mines, Investigations in Ore Dressing and Metallurgy, July to December 1934. 202 pages, tables, 16.5 × 25 cms. Ottawa, King's Printer, 1936.

U. S. Weather Bureau, Daily River Stages at River Gage Stations on the Principal Rivers of the United States, vol. XXXII for the year 1934. 159 pages, 23 × 28.5 cms. Washington, Government Printing Office, 1936. Price thirty-five cents.

National Advisory Committee for Aeronautics, Bibliography of Aeronautics, 1932. 295 pages, 18 × 26 cms. Washington, Government Printing Office, 1936. Price fifty cents. Technical Notes: No. 557, Considerations of the Take-Off Problem, by Edwin P. Hartman, 10 pages, figures. No. 558, The Performance of a DePalma Roots-Type Supercharger, by Oscar W. Schey and Herman H. Ellerbrock, Jr. 12 pages, figures. No. 559, Combustion-Engine Temperatures by the Sodium Line-Reversal Method, by Maurice J. Brevoort, 6 pages, illustrations. No. 560, A Comparison of Corrosion-Resistant Steel (18 Per Cent. Chromium—8 Per Cent. Nickel) and Aluminum Alloy (24St) by J. E. Sullivan, 12 pages. No. 561, Full-Scale Wind-Tunnel Tests to Determine a Satisfactory Location for a Service Pitot-Static Tube on a Low-Wing Monoplane, by John F. Parsons, 6 pages, figures. 5 pamphlets, 20 × 26 cms. Washington, Committee, 1936.

Technical Books of 1935, 23 pages, 11.5 × 17.5 cms. Brooklyn, Pratt Institute Free Library, 1936.

International Nickel Company of Canada, Ltd., Annual Report for the Year Ended December 31, 1935. 27 pages, 21 × 27 cms. Canada, 1936.

Universal Magnetism, A New Theory in Physics and Astronomy, 31 pages, 9 × 12 cms. Chicago, W. C. Ewert. Price $1.00.

Bell Telephone Laboratories, Monographs: B–893, Radio-Frequency Distributing Systems, by F. X. Rettenmeyer, 22 pages, illustrations. B–894, Eclipse Effects in the Ionosphere, by J. P. Schafer and W. M. Goodall, 5 pages, diagrams. B–895, On the Correlation of Radio Transmission with Solar Phenomena, by A. M. Skellett, 9 pages. B–900, Flutter in Sound Records, by T. E. Shea, W. A. MacNair and V. Subrizi, 11 pages, diagrams. B–901, Earth Resistivity and Geological Structure, by R. H. Card, 21 pages, diagrams. B–902, Acoustic Impedance of Small Orifices, by L. J. Sivian, 8 pages, diagrams. B–903, Some Bessel Function Expansions, by W. H. Wise, 7 pages. B–904, Transitions in Camphor and Chemically Related Compounds—I, by W. A. Yager and S. O. Morgan, 24 pages, diagrams. B–905, Transitions in Camphor and Chemically Related Compounds—II, by Addison H. White and S. O. Morgan, 28 pages, diagrams. B–906, Ultra-Short-Wave Propagation Over Land, by Chas. R. Burrows, Alfred Decino and Loyd E. Hunt, 29 pages, illustrations, tables. B–907, Uniformity in Photographic Development, by J. Crabtree, 10 pages, illustrations. B–908, Magnetic Alloys of Iron, Nickel and Cobalt in Communication Circuits,

by G. W. Elmen, 20 pages, illustrations, tables. B–910, Long-Wave Radio Phenomena Associated with Cessation of Sun's Rays, by Austin Bailey and A. E. Harper, 18 pages, diagrams, tables. B–911, The Present Status of Ferromagnetic Theory, by R. M. Bozorth, 29 pages, diagrams. B–912, The Corrosion of Metals —I, by R. M. Burns, 19 pages, diagrams. B–913, Magnetic Measurements at Low Flux Densities Using the A. C. Bridge, by Victor E. Legg, 24 pages, diagrams. B–914, Equivalence Theorems of Electromagnetics, by S. A. Schelkunoff, 21 pages, diagrams. 18 pamphlets, 15 × 23 cms. New York, Bell Laboratories, 1936.

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